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RESEARCH ARTICLE

Option-implied betas and the cross section of stock returns

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We investigate the cross-sectional relationship between stock returns and a number of measures of option-implied beta. Using portfolio analysis, we show that the method proposed by Buss and Vilkov (2012, *The Review of Financial Studies*, 2525, 3113–3140) leads to a stronger relationship between implied beta and stock returns than other approaches. However, using the Fama and MacBeth (1973, *Journal of Political Economy*, 8181, 607–636) cross-section regression methodology, we show that the relationship is not robust to the inclusion of other firm characteristics. We further show that a similar result holds for implied downside beta. We, therefore, conclude that there is no robust relation between option-implied beta and returns.

KEYWORDS
cross section, downside beta, option-implied beta, stock returns

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1 | INTRODUCTION

The capital asset pricing model (CAPM), developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966), predicts that the expected return of a stock should be a positive linear function of its market beta, and unrelated to all other characteristics of the stock. These predictions of the CAPM have been empirically tested in many studies.[1] However, these studies typically estimate the unobserved beta using historical data on stock returns. As noted by McNulty, Yeh, Schulze, and Lubatkin (2002), the use of historical stock returns to estimate market beta is problematic, since it leads to sensitivity to minor changes in the sample period used.

In an attempt to reduce the estimation error that arises from the use of historical data, a number of studies have developed estimators of market beta that exploit information about the covariance matrix of stock returns that is contained in option prices. French, Groth, and Kolari (1983; FGK) introduce a hybrid method to estimate market beta that combines an estimate of the correlation between the stock return and the market return from historical data with the ratio of stock-to-market implied volatility. Chang, Christoffersen, Jacobs, and Vainberg (2011; CCJV) use both option-implied skewness and volatility to estimate market beta. They find that the CCJV beta performs relatively well and can explain a sizeable proportion of cross-sectional variation in expected returns. Buss and Vilkov (2012; BV) compute option-implied beta using option-implied correlation and volatility. They find that in support of the CAPM, there is a monotonically increasing relation between BV beta and returns.

Buss and Vilkov (2012) compare their approach with both historical beta, and other option-implied betas, using tests based on portfolio sorting, and conclude that the BV beta performs best. In this paper, we investigate the robustness of these findings with respect to the inclusion of other firm-specific characteristics. We employ options on the S&P 500 index and its

[1]See, for example, Fama and French (1992), who find that the relation between market betas and average returns disappears during the more recent 1963–1990 period of U.S. stock return data even when beta is the only explanatory variable.

constituents to construct option-implied betas. Using both the portfolio sorting approach and the Fama and MacBeth (1973) regression approach, we compare the performance of four methods of estimating market beta: The historical beta, the FGK beta of French et al. (1983), the CCJV beta of Chang et al. (2011), and the BV beta of Buss and Vilkov (2012). We also develop several measures of option-implied downside betabasplie3hmatin37-282[(implied)-374b5(d)1.6(eta)-236m5(d)1.6methodaebotaownsi) where $\sigma_{i,t}^{Q}$ and $\sigma_{M,t}^{Q}$ are the option-implied volatility for stock i and the index, respectively, and $\rho_{i,M}$ is the correlation between historical stock and index returns.

2.3 | CCJV beta

Chang et al. (2011; CCJV) propose a one-factor model and assume zero skewness of the market return residual to propose a new market beta method by using both option-implied volatility and skewness. The CCJV implied beta is defined as

$$= \left(\frac{}{} \right)^{-} \frac{}{}$$

2.5 | Implied downside betas

We use three methods to estimate downside beta, namely, the historical approach, and approaches based on the FGK and BV implied betas.[3] For the historical downside beta, we follow the semivariance beta approach of Hogan and Warren (1972). The computation of historical downside betas is as follows:

$$\beta_\theta^{D\text{-His}} = \frac{E[r_i r_M | r_M < \theta]}{E[r_M^2 | r_M < \theta]}. \tag{8}$$

where the numerator is the second lower partial comoment between the excess return of stock i, r_i, and the excess market return, r_M, and measures the comovement between the stock and the market during market downturns. The threshold, θ is used to define the downside market. In this paper, we set θ to be the mean of the excess market return, r_M.

The principle for modeling implied downside beta is based on modeling downside correlations. Ang et al. (2002) decompose downside beta into a conditional correlation term and a ratio of conditional total volatility to conditional market volatility. The downside correlation is given by:

$$\rho_\theta^- = \text{corr}\{r_i, r_M\} = \frac{E[r_i r_M | r_M < \theta]}{\sqrt{E[r_i^2 | r_M < \theta] E[r_M^2 | r_M < \theta]}}. \tag{9}$$

Following Ang et al. (2002), we combine downside correlation and option-implied volatility to obtain implied downside beta. We substitute the historical correlation of the FGK beta in Equation 2 by the downside correlation in Equation 9 to obtain the FGK implied downside beta:

$$\beta_\theta^{D\text{-FGK}} = \rho_\theta^- \frac{\sigma_{i,t}^Q}{\sigma_{M,t}^Q}. \tag{10}$$

For the BV beta method, we use individual stock returns satisfying $[r_i | r_M < \theta]$ to calculate the physical downside correlations ρ_θ^- and then obtain the BV implied downside beta using Equations 4.

3 | DATA DESCRIPTION AND SAMPLE STATISTICS

3.1 | Data

We employ daily options on the S&P 500 index and its constituents from OptionMetrics for the period from January 1996 to April 2016, a total of 5,116 trading days. We extract the security ID, expiration date, call or put identifier, strike price, best bid, best offer, and implied volatility from the option price file. The sample includes both European and American options. For European option

$<\$3/8$. We also filter out quotes that do not satisfy standard no-arbitrage conditions. For calls, we require the bid price to be less than the spot price and the offer price to be at least as large as the spot price minus the strike price. For puts, we require the bid price to be less than the strike price and the offer price to be at least as large as the strike price minus the spot price. We eliminate in-the-money options because they are less liquid than out-of-the-money (OTM) and at-the-money options. We mitigate the effect of an early exercise premium on our estimations by eliminating put options with $K/$

preranked betas are estimated using previous 180-day (126-trading day) daily returns at the end of month t. These

TABLE 2 Portfolio analysis sorted by different beta methods

	Low	2	3	4	High	High–low	MR_p
Panel A. Historical beta							
Beta	0.48	0.75	0.95	1.18	1.72	1.24	–
vw-return	0.67	0.83	0.73	0.91	0.80	0.14	0.49
	(3.04)	(3.44)	(2.41)	(2.69)	(1.57)	(0.31)	–
ew-return	0.82	0.97	1.03	1.15	1.10	0.28	0.21
	(3.60)	(3.46)	(3.11)	(3.02)	(1.95)	(0.59)	–
Panel B. FGK beta							
Beta	0.41	0.64	0.79	0.96	1.32	0.91	–
vw-return	0.68	0.73	0.89	0.81	0.69	0.01	0.29
	(3.00)	(2.77)	(2.96)	(2.17)	(1.24)	(0.02)	–
ew-return	0.79	0.85	1.09	1.20	0.98	0.19	0.52
	(3.27)	(2.94)	(3.13)	(2.90)	(1.69)	(0.41)	–
Panel C. CCJV beta							
Beta	0.00	0.74	0.92	1.10	1.48	1.48	–
vw-return	0.85	0.76	0.81	0.71	0.76	−0.09	0.25
	(3.31)	(2.99)	(2.78)	(1.96)	(1.49)	(−0.25)	–
ew-return	1.23	0.98	1.09	0.85	0.73	−0.51	0.65
	(3.75)	(3.51)	(3.35)	(2.22)	(1.40)	(−1.62)	–
Panel D. BV beta							
Beta	0.66	0.86	1.01	1.19	1.62	0.96	–
vw-return	0.63	0.79	1.09	0.84	1.00	0.37	0.66
	(2.57)	(2.85)	(3.57)	(2.19)	(1.74)	(0.79)	–
ew-return	0.74	0.96	1.11	1.17	1.08	0.35	0.30
	(2.85)	(3.16)	(3.29)	(2.89)	(1.76)	(0.68)	–

Note. The five quintile portfolios are sorted by different betas over the sample period from January 1996 to April 2016. At the end of each month, we sort the stocks into quintiles based on their betas. The first portfolio then contains the stocks with the lowest market beta, while the last portfolio contains the stocks with the highest market beta. We then compute the value-weighted and equally weighted monthly returns over the next month for each quintile portfolio, month and beta methodology. The table reports the time-series average of betas and the value-weighted (vw-return) and equally weighted (ew-return) portfolio returns, as well as the high–low portfolio return spread, separately for each beta methodology. In addition, the table provides Newey-West (1987) *t* statistics for the high–low spread (shown in parentheses) to test whether the spread is significant or not. It also provides *P* values, obtained from time-series block bootstrapping, for the Patton and Timmermann (2010) monotonic relation (MR) test. The returns are expressed in percentages.

high–low return spread is 0.37% per month for the value-weighted portfolios and 0.35% per month for the equally weighted portfolios. Neither of these values is statistically significant at conventional levels observed from the Newey and West (1987) *t* statistics. Overall, the portfolio analysis shows that the BV beta gives the biggest high–low return spread compared with the other beta methods.

We perform a formal monotonic relation (MR) test of the risk-return relation, applying the nonparametric technique of Patton and Timmermann (2010). The results of the MR test, with *P* values obtained from time-series block bootstrapping, are shown in the last column of Table 2. If the high–low return spread is positive (negative), the null hypothesis of the MR test is that there is no relation or a weakly decreasing (increasing) relation between beta and returns, while the alternative hypothesis is that there is an increasing (decreasing) relation between beta and returns. All MR *P* values are greater than 10%, suggesting that there is no significant evidence to support the existence of a monotonically increasing relation between beta and returns.

To summarize, Table 2 shows that the relationship between the historical, FGK and BV betas, and stock returns is positive, but not statistically significant. The BV beta gives the biggest value-weighted and equally weighted return spread between the extreme portfolios, which is consistent with the findings of Buss and Vilkov (2012).

Figure 1 shows that all beta methods display a noisy beta-return relation across different quintiles for the value-weighted returns. For the value-weighted portfolios, the return difference between the extreme quintile portfolios is more pronounced for the BV and historical betas than for the FGK and CCJV beta methods. The return spread using BV beta is more pronounced than that using historical beta. The plot of the BV beta and

TABLE 3 Fama–MacBeth regressions for general betas

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8
Constant	0.77	0.61	1.42	0.37	1.29	1.28	1.47	1.28
	(2.70)	(2.02)	(4.26)	(0.92)	(1.69)	(1.67)	(1.96)	(1.67)
Historical	0.21				−0.05			
	(0.56)				(−0.16)			
FGK		0.62				0.14		
		(1.27)				(0.36)		
CCJV			−0.41				−0.40	
			(−2.28)				(−3.08)	
BV				0.53				0.22
				(1.06)				(0.43)
Size					−0.14	−0.15	−0.11	−0.14
					(−2.28)	(−2.44)	(−1.89)	(−2.32)
BM					0.02	0.02	0.02	0.02
					(0.32)	(0.28)	(0.33)	(0.26)
ivol					2.99	−1.14	6.15	−11.00
					(0.18)	(−0.07)	(0.35)	(−0.60)
VRP					0.06	−0.18	−0.21	−0.19
					(0.06)	(−0.15)	(−0.18)	(−0.13)
Momentum					0.03	0.06	0.19	0.11
					(0.07)	(0.18)	(0.52)	(0.32)
Illiquidity					0.59	0.57	0.35	0.54
					(1.74)	(1.68)	(0.92)	(1.62)
Lag return					−1.54	−1.59	−1.41	−1.62
					(−1.95)	(−2.01)	(−1.64)	(−2.05)
Return_m	−0.35	−0.22	1.51	−0.24				
					(−0.15)	(−0.10)	(0.61)	(−0.10)
Adj R^2 (%)	6.66	6.32	2.01	7.27	16.64	16.67	15.14	16.70

Note. The table shows the results for the Fama and MacBeth (1973) regression of monthly stock returns on betas and firm characteristics. The sample period is from January 1996 to April 2016. We report the average of coefficients and their t statistics (shown in parentheses) of the independent variables.

the value-weighted returns shows that the pattern is closest to linear compared with the historical, FGK, and CCJV betas. The equally weighted return for both the historical and BV beta methods displays a monotonically increasing risk-return relation.

4.2 | Fama–MacBeth regressions

We adopt the Fama and MacBeth (1973; FM) regression approach to further examine the risk-return relationship, and in particular, to explore its robustness to a wide range of firm-specific characteristics. The previous literature supports the existence of a firm size effect (Banz, 1981), a book-to-market effect (Basu, 1983), a momentum effect (Jegadeesh & Titman, 1993), an idiosyncratic volatility effect (Ang, Hodrick, Xing, & Zhang, 2006), a reversal effect (Jegadeesh, 1990; Lehmann, 1990), a maximum daily return effect (Bali, Cakici, & Whitelaw, 2011) and an illiquidity effect (Amihud, 2002). The calculation of firm size and book-to-market follows Fama and French (1992). We define the firm size (*size*) as the natural logarithm of the market capitalization from the previous day, where market capitalization is equal to the stock price multiplied by the number of shares outstanding. Book-to-market (*BM*) is the natural logarithm of book value to market value, where book value is the book value of common equity plus balance-sheet deferred taxes. Idiosyncratic volatility (*ivol*) is the standard deviation of the residuals from a regression of the excess stock return on the excess market return and the size (*SMB*) and book-to-market (*HML*) factors of Fama and French (1993), again using daily returns over the previous one year.[6] The momentum measure

[6]The daily market excess return, SMB, HML and the risk-free rate are taken from Kenneth French's website, http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/datalibrary.html.

(*Momentum*) is the cumulative daily stock return from month $t-12$ to month $t-1$. The illiquidity measure (*Illiquidity*) is the average of the ratio of the daily absolute return to the (dollar) trading volume over the previous year. Following Jegadeesh (1990) and Lehmann (1990), the reversal measure (*lag return*) is defined as the monthly return over the previous month. The maximum daily return (*return_m*) is defined as the maximum daily return over the previous month. As in Carr and Wu (2008), the variance risk premium (*VRP*) is defined as the difference between realized variance and option-implied variance:

$$VRP(t) = \sigma_P^2(t)\text{-}\sigma_Q^2(t) \tag{11}$$

where $\sigma_P^2(t)$ and $\sigma_Q^2(t)$ denote the realized and implied variances in month t, respectively.

Table 3 presents the results for the Fama and MacBeth (1973) regressions of stock returns on the different measures of beta and firm-specific characteristics. Models 1–4 include only beta in the Fama and MacBeth (1973) regression. When beta is the only independent variable, the coefficients of these betas are not significant except in the case of the CCJV beta, where the coefficient is actually negative. When firm-specific variables are included in the Fama and MacBeth (1973) regressions in Models 5–8, we find that beta still has no significant explanatory power for stock returns except for the CCJV beta, which again has a negative coefficient. Additionally, we find that size and lagged return are significantly and negatively related to stock returns.

TABLE 4 Portfolio analysis on implied downside betas

	Low	2	3	4	High	High–low	MR_p
Panel A. Historical downside beta							
Beta	0.48	0.76	0.96	1.19	1.72	1.24	–
vw-return	0.71	0.80	0.73	0.90	0.86	0.15	0.35
	(3.26)	(3.12)	(2.58)	(2.55)	(1.69)	(0.34)	–
ew-return	0.82	0.97	1.03	1.10	1.16	0.33	0.10
	(3.65)	(3.52)	(3.21)	(2.82)	(2.05)	(0.72)	–
Panel B. FGK downside beta							
Beta	0.24	0.47	0.62	0.77	1.07	0.83	–
vw-return	0.74	0.76	0.71	0.99	0.67	−0.07	0.88
	(3.16)	(2.84)	(2.34)	(2.80)	(1.30)	(−0.18)	–
ew-return	0.83	0.90	1.00	1.21	1.00	0.17	0.57
	(3.42)	(3.01)	(2.87)	(2.96)	(1.82)	(0.41)	–
Panel C. BV downside beta							
Beta	0.70	0.89	1.03	1.21	1.63	0.93	–
vw-return	0.64	0.90	0.86	0.89	1.07	0.43	0.12
	(2.66)	(3.33)	(2.71)	(2.27)	(1.69)	(0.78)	–
ew-return	0.79	0.92	1.09	1.06	1.19	0.39	0.27
	(3.25)	(3.07)	(3.28)	(2.54)	(1.92)	(0.76)	–

Note. The five quintile portfolios are sorted by downside betas over the sample period from January 1996 to April 2016. At the end of each month, we sort the stocks into quintiles based on their downside betas. The first portfolio then contains the stocks with the lowest market downside beta, while the last portfolio contains the stocks with the highest market downside beta. We then compute the value-Ta417fe 41,value

5 | OPTION-IMPLIED DOWNSIDE BETAS AND THE CROSS SECTION OF STOCK RETURNS

5.1 | Portfolio analysis

We sort the individual securities in the S&P 500 index into five groups at the end of each month by each of the four measures of downside beta. Portfolio 1 includes firms with the lowest downside betas and portfolio 5 contains firms with the highest downside betas. We then calculate the annualized value-weighted and equally weighted return for each beta method, for each portfolio in the next month. The procedure is repeated for all months. Table 4 provides a summary of the results. The table shows that the high–low return spread (the difference between the fifth and first portfolio returns) is positive for the historical, FGK, and BV downside beta methods in most cases. Taking the value-weighted returns as an example, the high–low return spread is 0.15% per month for the historical downside beta in Panel A, −0.07% per month for the FGK downside beta in Panel B and 0.43% per month for the BV downside beta in Panel C. The portfolio sorting method thus suggests that there is a positive relationship between the historical and BV downside betas, and stock returns, although in neither case is the difference statistically significant at conventional levels. As in the case of standard beta, the BV implied downside beta gives the biggest value-weighted and equally weighted high–low return spread between the extreme portfolios. From the MR test in the last column of Table 4, we find that all MR *P* values are greater than 10% for the value-weighted returns, suggesting that there is no monotonically increasing relation between downside beta and either value-weighted or equal-weighted returns.

Comparing the results in Table 4 with those in Table 2, we see that the BV implied downside beta performs better than the BV standard beta in terms of the positive and linear beta-return relation. More specifically, the BV downside beta gives an average high–low return spread of 0.43% per month for value-weighted returns, compared with a difference of 0.37% per month for the BV standard beta. The result that the BV downside beta outperforms the BV standard beta is consistent with published research (e.g., Ang et al., 2006; Post & Van Vliet, 2004). For instance, Post and Van Vliet (2004) find that the mean semivariance CAPM strongly outperforms the traditional mean-variance CAPM in terms of its ability to explain the cross section of U.S. stock returns.

TABLE 5 Fama–MacBeth regressions for implied downside beta

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Constant	0.77	0.74	0.48	1.33	1.21	1.23
	(2.74)	(3.04)	(1.13)	(1.76)	(1.62)	(1.60)
His(−)	0.21			−0.11		
	(0.56)			(−0.41)		
FGK(−)		0.52			0.05	
		(1.09)			(0.16)	
BV(−)			0.42			0.13
			(0.80)			(0.25)
Size				−0.13	−0.14	−0.13
				(−2.23)	(−2.28)	(−2.19)
BM				0.03	0.01	0.02
				(0.36)	(0.20)	(0.24)
ivol				3.55	0.45	−8.63
				(0.22)	(0.03)	(−0.45)
VRP				0.18	0.40	0.09
				(0.16)	(0.36)	(0.06)
Momentum				0.02	0.07	0.13
				(0.06)	(0.18)	(0.37)
Illiquidity				0.58	0.48	0.51
				(1.69)	(1.37)	(1.50)
Lag return				−1.61	−1.63	−1.72
				(−2.00)	(−1.97)	(−2.13)
Return max				−0.09	0.50	0.44
				(−0.04)	(0.21)	(0.19)
Adj R^2 (%)	6.44	4.68	7.15	16.46	15.97	16.32

Note. The table shows the results for the Fama and MacBeth (1973) regression of one-month ahead monthly stock returns on downside betas and firm-specific characteristics. The downside betas include the historical beta (His(−)), the FGK downside beta (FGK(−)), and the BV downside beta (BV(−)). The sample period is from January 1996 to April 2016. We report the coefficients and the *t* statistics (shown in parentheses) of the independent variables.
*, **, and *** denote the 10%, 5%, and 1% significance levels, respectively.

Figure 2 shows that the three downside beta methods display a noisy beta-return relation across different quintile portfolios for the value-weighted returns. For the value-weighted portfolios, the return difference between the extreme quintile portfolios is more pronounced for the BV downside beta than for the historical and FGK downside betas. The FGK downside beta gives the flattest beta-return relation, while the BV downside beta displays a relatively increasing risk-return relation. The equally weighted quintile portfolio returns for the historical and BV downside betas display a monotonically increasing risk-return relation.

5.2 | Fama–MacBeth regressions

We run the Fama and MacBeth (1973) monthly regression of stock returns on each of the three measures of downside beta, and include the same set of control variables, namely, firm size, book-to-market ratio, idiosyncratic volatility, the variance risk premium, momentum, lagged return, maximum daily return, and illiquidity. Table 5 presents the results of this analysis. Models 1–3 show the results for the Fama and MacBeth (1973) regression including only downside beta. For all three approaches, the coefficient on downside beta is positive but not significant. When the control variables are added in Models 4–6, the coefficient on downside beta remains insignificant in all three cases. The Fama and MacBeth (1973) regression approach therefore suggests that there is no significant relationship between returns and downside beta.

6 | CONCLUSION

Motivated by the earlier research of Buss and Vilkov (2012), this paper further investigates the relation between option-implied betas and stock returns. Consistent with the findings of Buss and Vilkov (2012), using portfolio analysis, we find that the BV beta outperforms other beta methods, giving the biggest positive high–low return spread. However, the return spread is not statistically significant.

We introduce measures of option-implied downside beta by combining the downside correlation of Ang et al. (2002) and the option-implied moments of Bakshi et al. (2003). When sorting stocks by downside beta, we again find that the BV downside beta yields the biggest high–low return spread. Moreover, a stronger beta-return relation is obtained using the BV downside beta than the BV standard beta.

We further explore the robustness of the beta-return relation using the Fama and MacBeth (1973) regression methodology. We find that using either standard beta or downside beta, the beta-return relationship is not significant nor is it robust to the inclusion of firm characteristics. We, therefore, conclude that there is no robust relation between option-implied beta and stock returns.

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APPENDIX

Estimation of risk-neutral moments

To compute risk-neutral model-free variance and skewness, we follow the formulas in Bakshi and Madan (2000) and Bakshi et al. (2003). Bakshi and Madan (2000) show that the continuum of characteristic functions of risk-neutral return density and the continuum of options are equivalent classes of spanning securities. Any payoff function with bounded expectation can be spanned by out-of-the-money (OTM) European calls and puts. Based on this insight, Bakshi et al. (2003) formalize a mechanism to extract the variance and skewness of the risk-neutral return density from a contemporaneous collection of OTM calls and puts. Their method relies on a continuum of strikes and does not incorporate specific assumptions about an underlying model. The two moments can be expressed as functions of payoffs on a quadratic and a cubic contract.

The prices of the quadratic and cubic contracts are given by

$$\text{Quad} = \int_S^\infty \frac{2\left(1-\ln\left(\frac{K}{S}\right)\right)}{K^2} C(\tau,K)\,\mathrm{d}K + \int_0^S \frac{2\left(1+\ln\left(\frac{S}{K}\right)\right)}{K^2} P(\tau,K)\mathrm{d}K, \tag{A.1}$$

$$\text{Cubic} = \int_S^\infty \frac{6\ln\left(\frac{K}{S}\right) - 3\left(\ln\left(\frac{K}{S}\right)\right)^2}{K^2} C(\tau,K)\,\mathrm{d}K - \int_0^S \frac{6\ln\left(\frac{K}{S}\right)+3\left(\ln\left(\frac{K}{S}\right)\right)^2}{K^2} P(\tau,K)\mathrm{d}K, \tag{A.2}$$

where S and K are the underlying stock price and strike price, respectively, and C and P are the call and put prices, respectively.

Using the prices of these contracts, the risk-neutral moments can be calculated as:

$$\text{VAR} = e^{r\tau}\,\text{Quad} - \mu^2, \tag{A.3}$$

$$\text{SKEW} = \frac{e^{r\tau}\text{Cubic} - 3\mu\, e^{r\tau}\text{Quad} + 2\mu^3}{\text{VAR}^{3/2}}, \tag{A.4}$$

where $\mu = e^{r\tau} - 1 - \frac{e^{r\tau}\text{Quad}}{2} - \frac{e^{r\tau}\text{Cubic}}{6}$.