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A time-varying copula approach for constructing a daily financial systemic stress index[I]

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ABSTRACT

This paper develops a financial systemic stress index (FSSI) for the US financial market. We propose a time-varying copula method to model the dependence structure among financial sectors in order to build a correlated financial stress model that can signal systemic financial risks. The copula method is preferable to the traditional approach, enabling the modeling of non-linear correlations. Our analyses show that the dependencies across banking, security, and forex markets are best modeled by Archimedian copulas. Finally, we conduct a Markov Switching Autoregressive (MS-AR) model for FSSI and identify high financial stress episodes taking place in 2008–2009, 2011 and 2020.

1. Introduction

Since the outbreak of the COVID-19 pandemic, global financial markets have been experiencing tremendous turmoil, which has manifested in volatile international capital movements and several stock market meltdowns of a scale and intensity comparable to those seen during the 2008 Global Financial Crisis (GFC). Fast forward to the present, wherein the financial markets appear to have returned to tranquility and the gradual reopening of the economy is anticipated; all in all, future global economic prospects remain highly uncertain. This is further complicated by the recent interest rate hikes by U.S. Federal Reserve to grapple with high inflation, which may hamper economic recovery and investor confidence.

Against such a backdrop of elevated financial uncertainty, this paper aims to introduce a copula-based daily financial systemic stress index (FSSI) that can be used, in time, to monitor the systemic financial stress of the US, taking into consideration time-varying dependence between different financial sectors. Our study is closely related to the strand of Financial Stress Index (FSI) literature pioneered by Illing and Liu (2006), who develop a continuous index of financial stress capable of measuring the intensity of crises for the Canadian financial system. Hollo, Kremer, and Lo Duca (2012) further propose an index of systemic financial stress that creatively models the synchronous nature of stresses across financial markets using a standard portfolio theory to the aggregation of the five market-specific stresses[1] into the composite indicator. We follow the index construction method of Hollo et al. (2012) to aggregate the stresses of banking, security, and forex markets into a single composite index using the pairwise time-varying

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[1] The five markets are money market, bond market, equity market, financial intermediaries, and foreign exchange market.

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correlations between the three sectors as weights. However, unlike previous studies which use Pearson correlation coefficient that can only capture linear interrelationships between variables, we estimate a time-varying dependence structure across sectors using various types of copula functions and use the Kendall's Tau correlations between sectors from the best fitted copula as bilateral weights between the market-specific stresses.

A copula is a multivariate cumulative distribution function (CDF) formed by uniform marginal distributions of each random variable. The most noteworthy strength of the copula method lies in its flexibility and effectiveness in characterizing joint extreme movements between variables since this allows for the estimation of joint distribution through the separate assessment of the copula and the marginals. This motivates for the increased application of the copula method within the financial domain Albulescu, Tiwari, and Ji (2020), Aloui, Hammoudeh, and Nguyen (2013), Bouri, Gupta, Lau, Roubaud, and Wang (2018), Nikoloulopoulos, Joe, and Li (2012), Zhang, Yan, and Tsopanakis (2018, such as,) to deal with the asymmetric dependence of financial returns, a phenomenon in which the comovement of returns is stronger during crisis periods than non-crisis periods (Patton, 2006). With the presence of asymmetric comovements, traditional methods such as the Pearson correlation, which assumes a normal distribution and the linear dependency of different financial returns, may lead to an incorrect estimation of financial stress. As such, copula methods are used to overcome this limitation of traditional methods when the normality assumption is violated. By incorporating market specific weights with copula-based correlations, our FSSI can take into account possible nonlinear dependencies between the different financial segments of a system, thus more accurately reflecting the systemic behavior of financial stress.

The contributions of this paper are twofold. First, our paper adds to the evolving stream of financial stability literature by developing a copula-based FSI as a measure for financial instability. This way, we are also able to investigate the potential nonlinear interdependence between financial sectors. Second, investors and policymakers may use the FSSI as an indicator for financial risk. For example, financial regulators can employ the FSSI as a real-time warning signal for financial crisis, while investors may use the FSSI to aid their investment decisions and adjust their leverage dependent on the state of financial stress.

This paper is organized as follows: In Section 2, we provide literature review of FSI studies. In Section 3, we introduce the market-specific stresses that constitute the FSSI and offer a preliminary inspection of cross-market stress dependence, which motivates for the copula-based construction method of the FSSI. Section 4 explains the methodology of the copula models adopted in this paper and proposes an algorithm for deriving time-varying dependence measures. Section 5 presents the estimation results of the copula models. Section 6 aggregates the market-specific stresses into the FSSI and identifies systemic financial risk states. In Section 7, we display results of several robustness tests. Section 8 gives a conclusion.

2. FSI literature

In the aftermath of the GFC, the volume of FSI studies has grown substantially. The existing FSI studies can roughly be divided into two main branches based on their research objectives.

2.1. Literature on FSI's application

One branch of FSI literature focuses on applying FSI to explore the transmission of financial stress during crises. There are three main types of financial stress transmission studied in the extant literature.

The first strand of literature studies the financial stress spillovers across different asset markets within a country. For example, Chau and Deesomsak (2014) examine FSI spillovers across the US equity, debt, banking, and forex markets during the period of 1991–2011 and uncover significant net spillover effects from debt and equity markets to others during financial crises. Apostolakis and Papadopoulos (2015) measure financial stress interdependence across the banking, securities and foreign exchange markets of G7 countries during the 1981–2009 period. The authors find that securities markets are the primary net transmitter of financial stress to other markets. Das, Kumar, Tiwari, Shahbaz, and Hasim (2018) use the causality-in-quantiles approach to investigate the causal relationships between FSI and the mean and variance of gold, crude oil, and stocks in the period of 1993–2017. Their results show the evidence of bilateral causality for gold and crude oil with FSI, while stocks have unilateral influence on FSI. Apostolakis, Floros, Gkillas, and Wohar (2021) evaluate connectedness of financial stress and economic policy uncertainty of 7 advanced countries with Brent oil prices over the period 2007–2020. According to their study, the spillover transmission is found to increase during the time of GFC, Brexit and COVID-19, and the oil prices are the net spillovers receiver from shocks stemming from financial stress and economic policy uncertainty.

The second strand of literature assesses the propagation of financial stress across countries. For example, Balakrishnan, Danninger, Elekdag, and Tytell (2011) provide a seminal study on financial stress transmission from advanced economies to emerging economies during the period of 1997–2009. Their findings suggest that the primary determinant of the magnitude of financial stress spillover is the financial linkages between advanced and emerging economies. Park and Mercado (2014) conduct a financial contagion analysis for 25 emerging market economies using data from 1992 to 2012 and substantiate domestic financial stress can be emanated from not only advanced economies, but also regional and nonregional emerging economies. Apostolakis and Papadopoulos (2014) examine financial stress spillovers across G7 economies for the 1981–2009 period. Their empirical results corroborate that the US is the primary FSI transmitter among G7 economies. MacDonald, Sogiakas, and Tsopanakis (2018) perform a multivariate GARCH model to assess the cross-covariance and spillover effects between the Eurozone financial markets for the period of 2001–2013. Their findings show that banking and money markets are the primary stress transmitters in Europe and that clustering effect exists as core economies are more exposed to spillover effects from their peers and the same holds for the periphery case. Elsayed and Yarovaya (2019) study the financial stress transmission of 8 MENA countries from 2015 to 2018, in which they conclude that the GFC dominates the Arab Spring in contributing to the financial spillovers across the MENA markets during the sample period.

The third strand of literature examines the stress transmission channels between the financial system and real economy. Cardarelli, Elekdag, and Lall (2011) identify financial stress episodes of 17 advanced economies using the countries' FSIs from 1980 to 2007 and analyzed under which circumstances these financial stress episodes lead to economic recessions. Their assessment results suggest that in contrast to stress that mainly affects securities and foreign exchange markets, financial turmoil caused by banking distress is more likely to result in deeper and longer downturns. Hubrich and Tetlow (2015) examine the interaction of financial stress with the real activity, inflation and monetary policy of the US using a Markov-switching VAR model. Their results show that switches to stress events jeopardize the real economic activities and conventional monetary policy is impotent during the stress events. Aboura and van Roye (2017) conduct similar study of interaction between financial stress and economic dynamics for the case of France and find that high financial stress events are significantly detrimental to economic activity but low financial stress episodes have negligible impact on economic dynamics.

2.2. Literature on FSI's construction approach

A second stream of FSI literature focuses on the technical aspect in terms of developing the construction approach of the composite index.

Among this branch, a group of studies explore the sets of components that should be embedded in the composite index to best represent the financial stress of a specific country. As mentioned above, Illing and Liu (2006) provide the pioneering work in this field. The authors propose a continuous measure of financial fragility that captures vulnerabilities in the banking, foreign exchange, debt and equity markets of the Canadian financial system. Hakkio, Keeton, et al. (2009) build measure of stress in the US financial system, named as the Kansas City Financial Stress Index (KCFSI), based on the principal component of 11 financial indicators. Nelson and Perli (2007) construct a financial fragility indicator for the US by combining three summary statistics, including index of normalized variables, rolling eight-week changes, and comovement indicator computed from 12 financial variables. Oet, Dooley, and Ong (2015) develop the Cleveland Financial Stress Index (CFSI) which captures stress of the US's financial system comprising the credit, funding, real estate, securitization, foreign exchange, and equity markets. Cevik, Dibooglu, and Kenc (2013) construct FSI for Turkey by aggregating Exchange Market Pressure Index, stock market volatility, bond spreads, default probability of banking sector, trade finance and growth rate of short term external debt. Kliesen, Smith, et al. (2010) develop a global financial stress indicator, which is called the OFR Financial Stress Index (OFR FSI) derived from 33 financial market indicators, such as yield spreads, valuation measures, and interest rates.

Another thread of research aims to refine the aggregation method for constructing the index. Hollo et al. (2012) note the importance of capturing the systemic nature of financial crises by encapsulating the simultaneous correlations across financial sectors when constructing the stress indicator for the overall financial system. In building their Composite Indicator of Systemic Stress (CISS), the authors apply ideas from the portfolio theory approach, which weights the stress of each individual financial segment according to its time-varying cross-correlations with the other segments. In this manner, the CISS would put more weight on circumstances when high stress prevails in several financial markets at the same time. This aggregation method has been adopted in other similar studies, including (Louzis & Vouldis, 2012) who developed an FSSI for Greece, and Duprey, Klaus, and Peltonen (2017), who constructed a monthly Country-Level Index of Financial Stress (CLIFS) for the EU countries.

3. Constructing a copula-based FSSI

3.1. Introducing components of market-specific stress

The country-level FSSI comprises three stress subindices for banking, security, and forex markets, all of which are measured by real-time daily market-based indicators. The choice of indicators in this paper follows (Apostolakis & Papadopoulos, 2014; Cardarelli et al., 2011; Melvin & Taylor, 2009). An overview of these subindices is provided as follows:

Banking market stress (I_{BK}). Stress in the banking market consists of three variables. First, in line with the standard capital asset pricing model, we measure the banking sector-specific shocks with the beta coefficient of the banking equity index, constructed as a 260-day rolling covariance of banking stock returns and the overall market returns relative to the variance of the overall market returns. Second, we include a TED or interbank spread, as given by the difference between 3-month interbank rates and yield on the Treasury Bill, to indicate the perceived credit risk in the banking sector. Third, the slope of the yield curve or inverted term spread, measured by the difference between the short- and long-term yields on government securities, is included to proxy a bank's profitability. Since a bank's profitability mainly hinges on its ability to convert short-term liabilities (demand deposits) into long-term assets (loans), a negative term spread poses a serious risk to the bank's income.

All the three indicators are standardized and averaged to obtain the banking market stress as :

$$I_{BK} = \frac{\text{beta} + \text{TED spread} + \text{Inverted term spread}}{3}$$

Security market stress (I_{SC}). Stress in the security market is constituted of three components. First, corporate bond spread as measured by the spread between yields on corporate bond and long-term government bond was used to proxy risk in the bond market. Second, we calculate inverted daily stock returns by multiplying $*1$ by the daily returns of S&P 500 composite index, such that a decline in stock returns is reflected as an increase in the security market stress. Third, we include the conditional stock market volatility derived from an exponentially weighted moving average (EWMA)[2] of daily squared returns.

The security market stress is the average of the three standardized components as shown below:

$$I_{SC} = \frac{\text{Corporate bond spread} + \text{Stock returns} + \text{Stock volatility}}{3}$$

Forex market stress (I_{FX}). Stress in the forex market is indicated by the time-varying volatility of forex returns, as proxied by standardized EWMA volatility of daily changes of nominal effective exchange rates.

$$I_{FX} = \text{Forex volatility}$$

There is a multitude of financial indicators which can be incorporated in FSSI. The selection of the foregoing seven indicators is due to two reasons. First, the primary aim of this study is to propose a copula-based construction method for FSSI that is replicable to other countries, including the emerging and developing economies where daily data availability is often lacking. Thus, we follow the existing cross-country studies of FSI (Apostolakis & Papadopoulos, 2014; Cardarelli et al., 2011; Melvin & Taylor, 2009) to adopt a uniform set of financial indicators that permits broad country coverage for any future replication. Second, as noted by Cardarelli et al. (2011), these seven indicators are selected as the principal financial time series that suffice to signal financial stress episodes of a country. Adding more will not bring much informative content, or worse still, may even contaminate the composite index with noisy signals. To check the sensitivity of our FSSI to the variable selection, we also conduct a robustness check by adding other financial indicators to our baseline FSSI. The alternative FSSI is largely similar to the baseline FSSI. More details can be found in Section 7.

3.2. A preliminary analysis of market-specific stresses

Using daily market data from the US that span the period from March 30, 2001, to March 5, 2021, we construct the stress placed on banking, security, and forex markets as plotted in Fig. 1. We observe that heightened stress occurred in the GFC period across all three markets. Specifically, the stress on both the banking and forex markets peaked at 2008, with a more abrupt pattern seen in the forex stress. During early 2020, when the COVID-19 pandemic first emerged, all three markets were hit by stress on a scale comparable to the GFC period. Most strikingly, the security market stress during the COVID-19 period surpassed that of the GFC period albeit a milder surge was found in the stress experienced by the banking and forex markets.

Table 1 presents the descriptive statistics of the three market-specific stresses. Both banking and security stresses have negative means, while forex stress has a positive mean. All three subindices are skewed to the right. The kurtosis of all market stresses exceeds three, indicating the presence of a heavy-tailed distribution for all three series. The non-zero skewness and excess kurtosis suggest that the financial stress is not normally distributed, a result further substantiated by the Jarque–Bera test. The significant ARCH test and Ljung–Box test confirm the existence of the ARCH effect and autocorrelation.

As a preliminary examination of the cross-market dependence structure, we adopt a graphical tool known as a Chi-plot, developed by Fisher and Switzer (1985, 2001). The Chi-plot inspects the dependency pattern between two series by comparing the empirical bivariate distribution against the null hypothesis of independence at each point in the scatter plot. Specifically, the bivariate distribution H and the two marginal distributions F and G of the two series $(\mathbf{x}, \mathbf{y})$ can be calculated using a nonparametric method as follows:

$$H_i = \frac{1}{n-1} \sum_{j \neq i} I(x_j \leq x_i, y_j \leq y_i),$$

$$F_i = \frac{1}{n-1} \sum_{j \neq i} I(x_j \leq x_i),$$

$$G_i = \frac{1}{n-1} \sum_{j \neq i} I(y_j \leq y_i).$$

where $I(A) = 0, 1$ according to whether A is false or true.

Then, the Chi-plot is generated by creating a scatter plot for each pair of indices (λ_i, χ_i) based on:

$$\chi_i = \frac{H_i - F_i G_i}{\sqrt{F_i(1 - F_i)G_i(1 - G_i)}}$$

and

$$\lambda_i = 4\,\text{sign}(\tilde{F}_i \tilde{G}_i)\,\max(\tilde{F}_i^{\,2}, \tilde{G}_i^{\,2})$$

where $\tilde{F}_i = F_i - \frac{1}{2}$, $\tilde{G}_i = G_i - \frac{1}{2}$ for $i \in \{1, \ldots, n\}$. To eliminate outliers, only the points that satisfy $|\lambda_i| < 4\left(\frac{1}{n-1} - \frac{1}{2}\right)^2$ are included in the display.

The parameter λ_i is a measure of the distance of the sample point (x_i, y_i) from the center of the dataset, and the value of χ_i is a measure of the distance of the joint distribution H to the distribution of independent pairs of random variables $(\mathbf{x}, \mathbf{y})$. Since

[2] A 60-day rolling window is adopted, and the decay factor λ is set as 0.94 following the parameter setting stipulated by the JP Morgan RiskMetrics database.

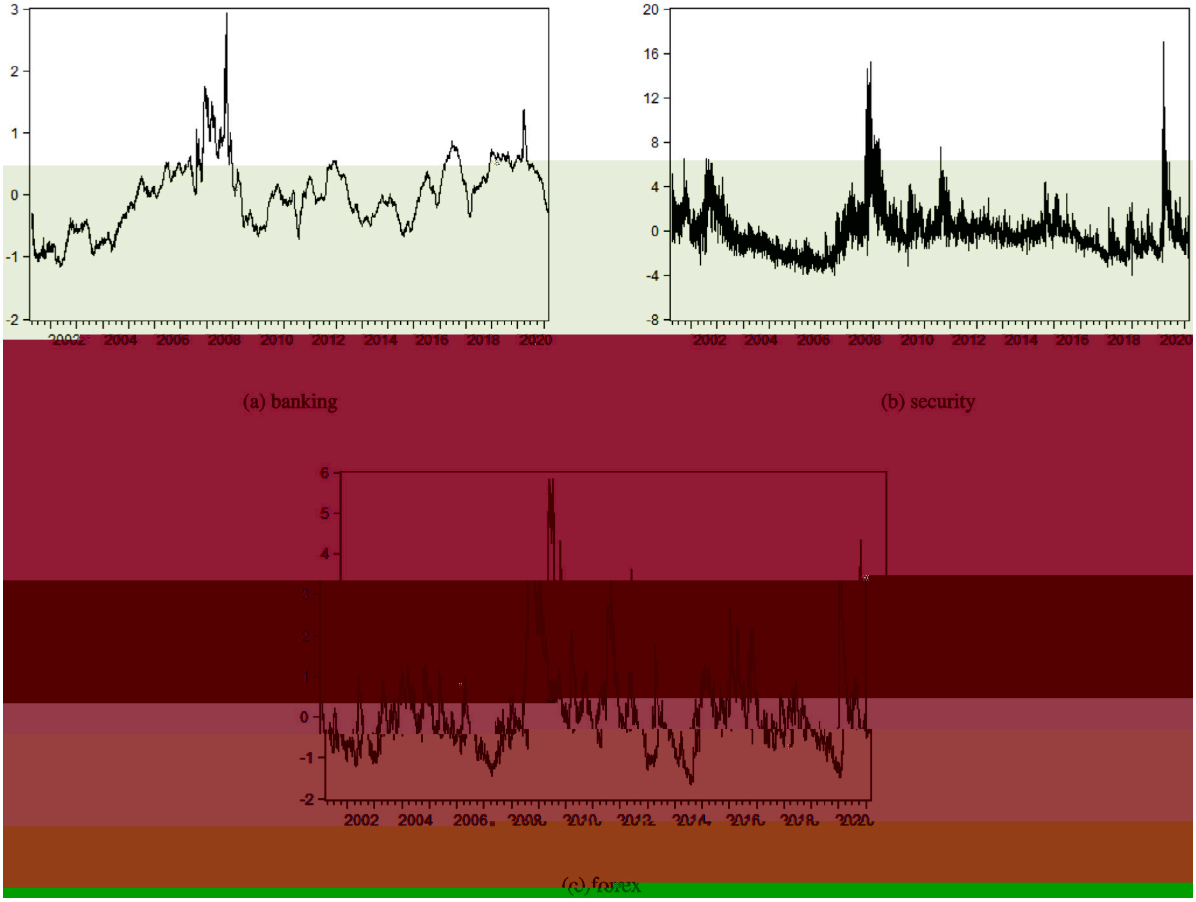

Fig. 1. Market-specific stress subindices.

Table 1
Descriptive statistics.

	Bank	Security	Forex
Mean	*0.0006	*0.0379	0.1005
Maximum	2.9356	17.0249	5.8240
Minimum	*1.1725	*4.0928	*1.6411
Std. Dev.	0.5504	2.0706	1.0262
Skewness	0.4222	1.9181	1.9486
Kurtosis	3.8643	10.4288	9.2145
Jarque–Bera	316***	15149***	11661***
ARCH test (5)	47460***	1089***	36465***
Ljung–Box test (5)	25657***	15108***	24597***
Observations	5201	5201	5201

Notes: *, **, and *** denote significance at the 10, 5 and 1% level.

$H_i = F_i G_i$ for all i under independence, values of χ_i that situate far from zero (beyond the 95 percent confidence band) imply a departure from independence.

Fig. 2 illustrates Chi-plots for each pair of market stresses. All pairs of subindices exhibit heavy-tail dependence, given that most distribution points fall outside the confidence bands with obvious curvature. Specifically, the dependencies are negative between the banking and security markets, and between the banking and forex markets. Meanwhile, positive dependence is found between the security and forex markets.

The Chi-plots provide a preliminary analysis of nonlinear dependence structures across market stresses and motivate for the use of the copula method to model such nonlinear dependencies.

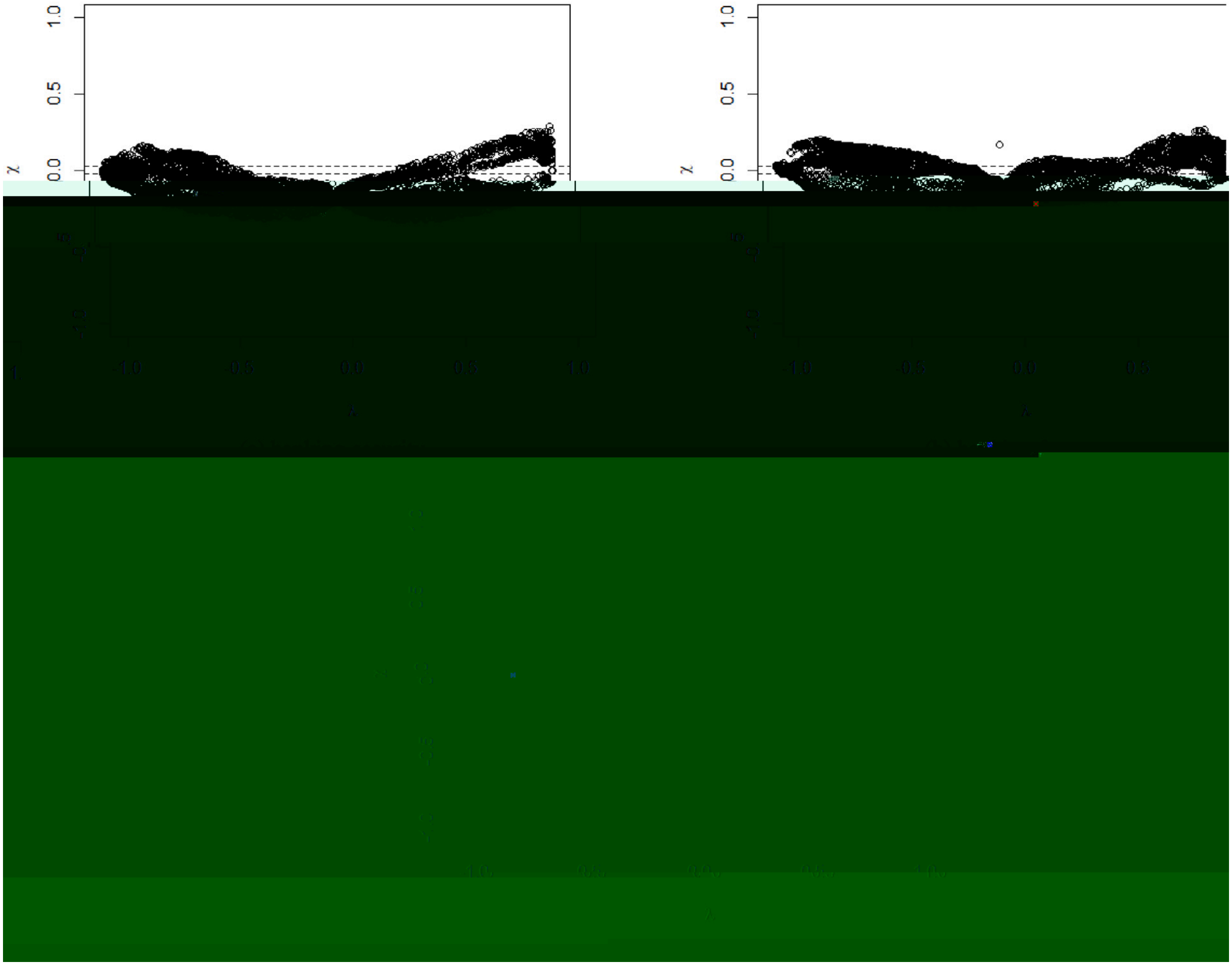

Fig. 2. Chi-plots illustrated by each pair of subindices.

3.3. A portfolio based aggregation approach to FSSI construction

Following the portfolio theory approach adopted by Hollo et al. (2012), the three market subindices are aggregated to a single composite indicator of system risk, namely, an FSSI, by weighting each component according to the pairwise cross-market correlations. In this setting, the overall financial stress would be intensified in the face of increased market co-movements. In contrast, lower cross-market linkages would mitigate any diversifiable risk, resulting in a lower risk of overall financial stress.

The three-market FSSI is computed as:

$$FSSI_t = \mathbf{I_t} \; C_t \; \mathbf{I_t}^{\prime} \tag{1}$$

where $\mathbf{I_t} = (I_{BK}, I_{SC}, I_{FX})$ and C_t is the matrix of the time-varying cross-correlations of the subindices:

$$C_t = \begin{bmatrix} 1 & \tau_{BK,SC,t} & \tau_{BK,FX,t} \\ \tau_{BK,SC,t} & 1 & \tau_{SC,FX,t} \\ \tau_{BK,FX,t} & \tau_{SC,FX,t} & 1 \end{bmatrix}$$

The cross correlations $\tau_{i,j,t}$ are Kendall's Tau coefficient, a measure of rank correlation, corresponding to the best fitted time-varying copula estimated in the next section.

4. Copula methodology

To model and analyze the dependence structure among the banking, security, and forex sectors, we employ a copula analysis.

Copula methods are widely used to measure the dependence structures (joint distribution) of two or more variables. This is based on the Sklar's theorem (1959), which states that a multivariate distribution can be expressed in terms of univariate marginal distribution functions and a copula function. By estimating the marginals and copulas separately, one can relax the assumptions of normality and linear correlation when constructing the multivariate distribution. In addition, the left and right tail dependencies can be measured using asymmetric copula families, such as the Archimedean copula, circumventing the assumption of symmetric tail dependence that goes with a normal distribution function.

An n-dimensional copula $c(\mu_1, \mu_2, \dots \mu_n)$ is defined as a distribution function on $I^n = [0,1]^n$ with standard uniform marginal distributions (Tsay, 2013). According to Sklar's theorem, every n-dimensional distribution of a random vector, $X = (x_1, x_2, \dots, x_n)$ with marginals $F_1(x_1), F_2(x_2), \dots, F_n(x_n)$ can be written as follows:

$$F(x_1, x_2, \dots, x_n) = C\left(F_1(x_1), F_2(x_2), \dots, F_n(x_n)\right),$$

where C is a copula that is uniquely determined on I^n for the distribution F with absolutely continuous margins as:

$$C(\mu_1, \mu_2, \dots, \mu_n) = F\left(F_1^{-1}(\mu_1), F_2^{-1}(\mu_2), \dots, F_n^{-1}(\mu_n)\right).$$

There are various families of copulas in the existing literature, including elliptical, Archimedean, Fréchet, Farlie–Gumbel–Morgenstern, Extreme Value, Asymmetric Logistic Model, and the Convex Combinations copulas. Among these copula families, the elliptical and Archimedean copulas are the two most widely used copulas in finance and economic application (Yeap, Lean, Sampid, & Hasim, 2020). In this paper, we focus on the Gaussian and Student's t copulas from the elliptical family, as well as Frank, Clayton, Gumbel, and Joe copulas from the Archimedean family. The Gaussian distribution is a widespread distribution in financial modeling, while the Student's t distribution is used to model extreme symmetric tail dependence. The four copulas chosen from the Archimedean family are also commonly used in financial engineering due to their various unique features. Specifically, the Frank copula is able to describe tail independence, while the Clayton, Gumbel, and Joe copulas are able to capture asymmetric dependence structures.

In the following, we present the functional forms of the selected copulas as mentioned above.

(i) The Gaussian copula. In n-dimension, the Gaussian copula can be expressed as:

$$C(\mu_1, \mu_2, \dots, \mu_n; \Sigma) = \Phi_n\left(\Phi^{-1}(\mu_1), \Phi^{-1}(\mu_2), \dots, \Phi^{-1}(\mu_n)\right) \tag{2}$$

where Φ_n and Φ^{-1} are the standard multivariate Gaussian and the inverse of the standard univariate Gaussian. Σ is the linear correlation matrix for the multivariate normal distribution. From Eq. (2), the bivariate copula and its density can be expressed as:

$$C(\mu_1, \mu_2; \rho) = \Phi_n\left(\Phi^{-1}(\mu_1), \Phi^{-1}(\mu_2); \rho\right)$$

$$c(\mu_1, \mu_2; \rho) = \frac{1}{\sqrt{1-\rho^2}} exp\left(-\frac{1}{2(1-\rho^2)}\left(x_1^2 - 2\rho x_1 x_2 + x_2^2\right)\right)$$

where x_1 and x_2 are random variables and ρ is their linear correlation coefficient. For $\rho = 1$ and $\rho = -1$, the copulas are comonotonic and counter-monotonic, respectively. For $\rho = 0$, the copula is independent.

Tail dependence is the measurement of dependence at the extreme quantiles. Upper tail dependence λ_U and lower tail dependence λ_L are defined as follows:

$$\lambda_U = \lim_{\zeta\to 1^-} P\left\{\mu_1 > F_1^{-1}(\zeta) \mid \mu_2 > F_2^{-1}(\zeta)\right\} = \lim_{\zeta\to 1^-} \frac{1 - 2\zeta + C(\zeta, \zeta)}{1-\zeta}$$

$$\lambda_L = \lim_{\zeta\to 0^+} P\left\{\mu_1 \le F_1^{-1}(\zeta) \mid \mu_2 \le F_2^{-1}(\zeta)\right\} = \lim_{\zeta\to 0^+} \frac{C(\zeta, \zeta)}{\zeta}$$

where $F_1^{-1}(\zeta)$ and $F_2^{-1}(\zeta)$ are the marginal quantile functions of the two variables μ_1 and μ_2 at the level ζ. Since the Gaussian copula exhibits no asymmetric behavior, both lower and upper tail dependencies are equal to zero ($\lambda_L = \lambda_U = 0$).

(ii) The Student's t copula. In n-dimension, the Student's t copula can be expressed as:

$$C(\mu_1, \mu_2, \dots, \mu_n; \Sigma, \upsilon) = T_\upsilon\left(t_{\upsilon_1}^{-1}(\mu_1), t_{\upsilon_2}^{-1}(\mu_2), \dots, t_{\upsilon_n}^{-1}(\mu_n)\right) \tag{3}$$

where T_υ and $t_{\upsilon_1}^{-1}$ are the standard multivariate Student's t and the inverse of the standard univariate Student's t with υ and υ_1 degrees of freedom, respectively. Σ is the linear correlation matrix for the multivariate Student's t distribution. From Eq. (3), the bivariate Student's t copula and its density can be expressed as:

$$C(\mu_1, \mu_2; \rho, \upsilon) = T_\upsilon\left(t_{\upsilon_1}^{-1}(\mu_1), t_{\upsilon_2}^{-1}(\mu_2); \rho, \upsilon\right)$$

$$c(\mu_1, \mu_2; \rho, \upsilon) = \frac{K}{\sqrt{1-\rho^2}}\left(1 + \frac{1}{V(1-\rho^2)}\left(\xi_1^2 - 2\rho\xi_1\xi_2 + \xi_2^2\right)\right)^{\frac{V+2}{2}} \left[\left(1+\upsilon^{-1}\xi_1^2\right)\left(1+\upsilon^{-1}\xi_2^2\right)\right]^{\frac{V+2}{2}}$$

where $\xi_i = t_{\upsilon_i}^{-1}(\mu_i)$ and $K = \Gamma\left(\frac{\upsilon}{2}\right)\Gamma\left(\frac{\upsilon+1}{2}\right)^{-2}\Gamma\left(\frac{\upsilon}{2}+1\right)$. ρ is the linear correlation coefficient of the bivariate Student's t distribution with υ degree of freedom. The Student's t copula is symmetric in tail dependence, and it approximates the Gaussian copula when the degree of freedom is large. The coefficient of tail dependence can be expressed as:

$$\lambda_L = \lambda_U = 2t_{\upsilon+1}\left(\frac{\sqrt{\upsilon+1}\sqrt{1-\rho}}{\sqrt{1+\rho}}\right)$$

(iii) The Frank copula. In n-dimension, the Frank copula can be expressed as:

$$C.\mu_1, \mu_2, \S\ , \mu_n; \theta/ = *\frac{1}{\theta} In\left(1 + \frac{\sum_{i=1}^{n} exp.*\theta\mu_i/ * 1}{exp.*\theta/ * 1}\right) \tag{4}$$

where θ is the copula parameter that takes any real value from $*\varnothing$ to $\varnothing$. This copula can capture both positive and negative dependence of the random variables. The copula is independent when $\theta = 0$, comonotonic when θ approaches $\varnothing$, and counter-monotonic when θ approaches $*\varnothing$. From Eq. (4), the bivariate Frank copula and its density can be expressed as:

$$C.\mu_1, \mu_2; \theta/ = *\frac{1}{\theta} In\left(1 + \frac{.exp.*\theta\mu_1/ * 1/.exp.*\theta\mu_2/ * 1/}{exp.*\theta/ * 1}\right)$$

$$c.\mu_1, \mu_2; \theta/ = \frac{\theta exp.*\theta.u_1 + u_2//.exp.*\theta/ * 1/}{[exp.*\theta.u_1 + u_2// * exp.*\theta u_1/ * exp.*\theta u_2/ + exp.*\theta/]^2}$$

The Frank copula does not have lower and upper tail dependence $.\lambda_L = \lambda_U = 0/$ like the Gaussian copula. This copula is more suitable for capturing the dependency of variables that are weak in tail dependence.

(iv) The Gumbel copula. In n-dimension, the Gumbel copula can be expressed as:

$$C.\mu_1, \mu_2, \S\ , \mu_n; \delta/ = exp\left(* \sum_{i=1}^{n} .*.ln.u_i//^{\delta}/^{1\_\delta}\right) \tag{5}$$

The Gumbel copula parameter takes values from range of one to infinity and it can only capture positive dependence. The copula is independent when $\delta = 1$ and comonotonic when $\delta \rightarrow \varnothing$. From Eq. (5), the bivariate Gumbel copula and its density can be expressed as:

$$C.\mu_1, \mu_2; \delta/ = exp\left\{ * \left([*ln.\mu_1/]^{1\_\delta} + [*ln.\mu_2/]^{1\_\delta}\right)^{\delta}\right\}$$

$$c.\mu_1, \mu_2; \delta/ = .A + \delta * 1/A^{1*\delta} exp.*A/.u_1u_2/^{*1}.*lnu_1/^{\delta*1}.*lnu_2/^{\delta*1}$$

where $A = [\left(*ln.\mu_1/\right)^{\delta}\left(*ln.\mu_2/\right)^{\delta}]^{1\_\delta}$. As the Gumbel copula is an asymmetric copula that can only capture the upper heavy tail, there is no lower tail for the Gumbel copula ($\lambda_L = 0$). The upper tail is $\lambda_U = 2 * 2^{1\_\delta}$ for all δ larger or equal to one.

(v) The Clayton copula. In n-dimension, the Clayton copula can be expressed as:

$$C.\mu_1, \mu_2, \S\ , \mu_n; \alpha/ = \left(* \sum_{i=1}^{n} u_i^{*\alpha} * n + 1\right)^{\frac{1}{\alpha}} \tag{6}$$

where α is the Clayton parameter from the range of zero to infinity and n is the number of random variables. The copula is independent when $\alpha \rightarrow 0$ and comonotonic when $\alpha \rightarrow \varnothing$. From Eq. (6), the bivariate Clayton copula and its density can be expressed as:

$$C.\mu_1, \mu_2; \alpha/ = .u_1^{\alpha} + u_2^{\alpha} * 1/^{\frac{1}{\alpha}}$$

$$c.\mu_1, \mu_2; \alpha/ = .1 + \alpha/\left(u_1^{*\alpha} + u_2^{*\alpha} * 1\right)^{*\frac{1}{\alpha}*2}.u_1u_2/^{\alpha*1}$$

As the Clayton copula is an asymmetric copula that can only capture the lower heavy tail, there is no upper tail for the Clayton copula $.\lambda_U = 0/$. The lower tail is $\lambda_L = 2^{*1\_\alpha}$ for all α larger than zero.

(vi) The Joe copula. In n-dimension, the Joe copula can be expressed as:

$$C.\mu_1, \mu_2, \S\ , \mu_n; \beta/ = 1 * \left(1 * \prod_{i=1}^{n}\left(1 * .1 * \mu_i/^{\beta}\right)\right)^{1\_\beta} \tag{7}$$

where β is the Joe parameter from the range of one to infinity. The copula is independent when $\beta \rightarrow 1$ and comonotonic when $\beta \rightarrow \varnothing$. From Eq. (7), the bivariate Joe copula and its density can be expressed as:

$$C.\mu_1, \mu_2; \beta/ = 1 * \left[.1 * \mu_1/^{\beta} + .1 * \mu_2/^{\beta} * .1 * \mu_1/^{\beta}.1 * \mu_2/^{\beta}\right]^{1\_\beta}$$

$$c.\mu_1, \mu_2; \beta/ = \frac{\left(\left(1 * .1 * \mu_1/^{\beta}\right).1 * \mu_2/^{\beta} + .1 * \mu_1/^{\beta}\right)^{\frac{1}{\beta}}\left(\left(1 * .1 * \mu_2/^{\beta}\right).1 * \mu_1/^{\beta} + \beta + .1 * \mu_2/^{\beta} * 1\right)}{[.1 * \mu_1/.1 * \mu_2/]^{1*\beta}\left(\left[.1 * \mu_2/^{\beta} * 1\right].1 * \mu_1/^{\beta} * .1 * \mu_2/^{\beta}\right)^2}$$

The Joe copula is similar to the Gumbel copula, which can only capture upper tail dependency. The upper tail of the Joe copula is $\lambda_U = 2 * 2^{1\_\beta}$ for all β larger or equal to one.

One of the limitations of the Clayton, Gumbel, and Joe copulas is that they can only measure positive dependence but not negative dependence. However, in financial data negative dependencies occur between variables. To overcome this limitation, we rotate the Clayton, Gumbel, and Joe copulas for 90ý, 180ý, and 270ý. A more detailed explanation of rotated copulas can be found in Sriboonchitta, Nguyen, Wiboonpongse, and Liu (2013).

A copula can be rotated 180ý (also known as a survival copula) using the following equations:

$$C^{**}.\mu_1, \mu_2/ = \mu_1 + \mu_2 * 1 + C.1 * \mu_1, 1 * \mu_2/$$

$$c^{**}.\mu_1, \mu_2/ = c.1 * \mu_1, 1 * \mu_2/$$

The 90ý rotated copula and its density can be expressed as:

$$C^{*+}.\mu_1, \mu_2/ = \mu_1 * C.1 * \mu_1, \mu_2/$$

$$c^{*+}.\mu_1, \mu_2/ = c.1 * \mu_1, \mu_2/$$

Finally, the 270ý rotated copula and its density can be written as:

$$C^{+*}.\mu_1, \mu_2/ = \mu_1 * C.\mu_1, 1 * \mu_2/$$

$$c^{+*}.\mu_1, \mu_2/ = c.\mu_1, 1 * \mu_2/$$

The static copula models as presented above cannot capture the dynamic dependence structure among variables. However, Lu, Lai, and Liang (2014) show that the performance of time-varying copulas is always superior to that of static copulas. Patton (2006) proposes a time-varying copula model by allowing the parameter to evolve over time using transformations of the lagged data and an autoregressive term. This method is also known as the autoregressive moving average (ARMA) process. Nonetheless, as mentioned earlier, the Gumbel, Clayton, and Joe copulas from the Archimedean family can reflect only positive dependence and not negative dependence, namely, the range of Kendall's Tau for these Archimedian copulas is in $[0, 1]$. The ARMA process proposed by Patton (2006) cannot capture negative dependence and is not realistic for our purpose of constructing a FSI. In order to capture both time-varying positive and negative dependence structures, we apply the window rolling method and propose a novel algorithm 1 as shown below, such that the time-varying Kendall's Tau series from the Gumbel, Clayton, and Joe copulas can take values in the range of $[*1, 1]$.

Algorithm 1: Time-varying copula

Result: To obtain Kendall's Tau with a range of $[*1, 1]$
1. Set a 500 day-long rolling window for the two selected variables $.\mu_1, \mu_2/$;
2. Fit the data into the selected copulas (Gumbel, Clayton, and Joe), and rotate the copulas 90ý, 180ý, and 270ý;
3. The best model is selected from the copulas and the rotated copulas based on Akaike information criterion(AIC);
4. The parameter obtained from the best fitted copula is converted into Kendall's Tau;
5. Repeat steps 1 to 4 until a series of Kendall's Tau is obtained;

5. Copula results

5.1. Static copula

The estimation results of static copula models including the corresponding Kendall's Tau coefficients are presented in Table 2. For the Clayton, Gumbel, and Joepula

Table 2
Static copula.

		Bank-sec	Bank-fx	Sec-fx			Bank-sec	Bank-fx	Sec-fx
Gaussian	Φ	*0.119<<<	0.031<<	0.364<<<	Frank	θ	*1.149<<<	*0.330<<<	1.942<<<
		(0.014)	(0.014)	(0.011)			(0.084)	(0.084)	(0.087)
	τ	*0.076	0.020	0.237		τ	*0.125	*0.036	0.203
	AIC	*72.181	*2.923	*735.309		AIC	*184.441	*13.567	*498.520
Student's t	T	*0.158<<<	0.011	0.339<<<					
		(0.015)	(0.017)	(0.014)					
	v	11.229<<<	30.000	8.109					
		(1.676)		(1.384)					
	τ	*0.101	0.007	0.220					
	AIC	*126.234	*5.268	*771.311					
Clayton	α	0.000	0.027<<	0.313<<<	*R180*	α	0.065<<<	0.059<<<	0.593<<<
		(0.023)	(0.014)	(0.020)			(0.012)	(0.013)	(0.023)
	τ	0.000	0.013	0.135		τ	0.031	0.029	0.229
	AIC	2.344	*2.123	*312.520		AIC	*30.966	*22.047	*969.866
R90	α	*0.106<<<	*0.053	*0.000	*R270*	α	*0.145<<<	*0.000	*0.000
		(0.020)	(0.035)	(0.032)			(0.017)	(0.015)	(0.020)
	τ	*0.050	*0.026	0.000		τ	*0.068	0.000	0.000
	AIC	*32.842	*8.188	2.358		AIC	*64.271	2.189	2.438
Gumbel	δ	1.019<<<	1.026<<<	1.305<<<	*R180*	δ	1.000<<<	1.000<<<	1.228<<<
		(0.005)	(0.006)	(0.013)			(0.010)	(0.008)	(0.013)
	τ	0.018	0.026	0.234		τ	0.000	0.000	0.185
	AIC	*24.519	*25.833	*986.554		AIC	2.590	2.029	*459.051
R90	δ	*1.090<<<	*1.000<<<	*1.000<<<	*R270*	δ	*1.061<<<	*1.000<<<	*1.000<<<
		(0.011)	(0.009)	(0.013)			(0.011)	(0.014)	(0.014)
	τ	*0.083	0.000	0.000		τ	*0.058	0.000	0.000
	AIC	*80.483	2.216	2.858		AIC	*30.211	2.003	2.776
Joe	β	1.057<<<	1.042<<<	1.461<<<	*R180*	β	1.000<<<	1.000<<<	1.218<<<
		(0.009)	(0.008)	(0.021)			(0.014)	(0.011)	(0.017)
	τ	0.032	0.024	0.206		τ	0.000	0.000	0.111
	AIC	*75.223	*38.726	*1002.975		AIC	2.465	2.032	*220.423
R90	β	*1.104<<<	*1.000<<<	*1.000<<<	*R270*	β	*1.015<<<	*1.000<<<	*1.000<<<
		(0.015)	(0.014)	(0.018)			(0.017)	(0.014)	(0.019)
	τ	*0.057	0.000	0.000		τ	*0.009	0.000	0.000
	AIC	*46.194	2.243	2.516		AIC	1.226	2.003	2.421

Notes: Standard errors in parentheses. *, **, and *** denote significance at the 10, 5, and 1% level. R90, R180, and R270 stand for the rotated 90ý, 180ý, and 270ý models, respectively. The underlined value is the lowest AIC among the copula and rotated copula. The bold value is the best copula selected among the copula model base on the lowest AIC. Bank, sec, and fx represent the banking, security, and forex sectors, respectively.

Table 3
Tail dependency.

Copula	Banking–security		Banking–forex		Security–forex	
	Lower	Upper	Lower	Upper	Lower	Upper
Student's t	1.4186 • 10^{*3}	1.4186 • 10^{*3}	5.0300 • 10^{*6}	5.0300 • 10^{*6}	6.2658 • 10^{*2}	6.2658 • 10^{*2}
Clayton	0.0000	0	6.1300 • 10^{*12}	0	1.0895 • 10^{*1}	0
Clayton R180	0	2.1800 • 10^{*5}	0	7.6700 • 10^{*6}	0	3.1098 • 10^{*1}
Gumbel	0	2.5047 • 10^{*2}	0	3.5174 • 10^{*2}	0	2.9900 • 10^{*1}
Gumbel R180	2.3416 • 10^{*4}	0	2.3416 • 10^{*4}	0	2.4112 • 10^{*1}	0
Joe	0	7.3367 • 10^{*2}	0	5.5541 • 10^{*2}	0	3.9265 • 10^{*1}
Joe R180	2.3416 • 10^{*4}	0	2.3416 • 10^{*4}	0	2.3320 • 10^{*1}	0
Average	4.7174 • 10^{*4}	2.4964 • 10^{*2}	1.1834 • 10^{*4}	2.2682 • 10^{*2}	1.6148 • 10^{*1}	2.6632 • 10^{*1}

Notes: The average is calculated by adding all the tail coefficients and dividing by four.

5.2. Time-varying copula

As mentioned in the previous section, a static copula cannot reflect the dynamic behavior of the dependence structures of market-specific stress. Therefore, we apply a 500 day-long rolling window time-varying copula with the algorithm 1 to model the dependence structures among the subindices.[4] The best model is selected based on the average AIC as presented in Table 4. The Joe copula is the

[4] As a robustness check, we also apply a 400 and 600 day-long rolling window. Detailed results are available upon request.

Table 4
Average AIC for time-varying copula.

Copula	Banking–security	Banking–forex	Security–forex
Gaussian	*113.6041	*93.3907	*196.6471
Student t	*112.8537	*94.7397	*198.7838
Clayton	*157.2472	*127.5121	*218.7487
Gumbel	*129.1592	*107.2263	*213.0850
Frank	*104.5676	*92.5832	*172.3549
Joe	*157.8356	*127.3517	*217.2937

Notes: The bold value is the best fitted copula model based on the lowest AIC.

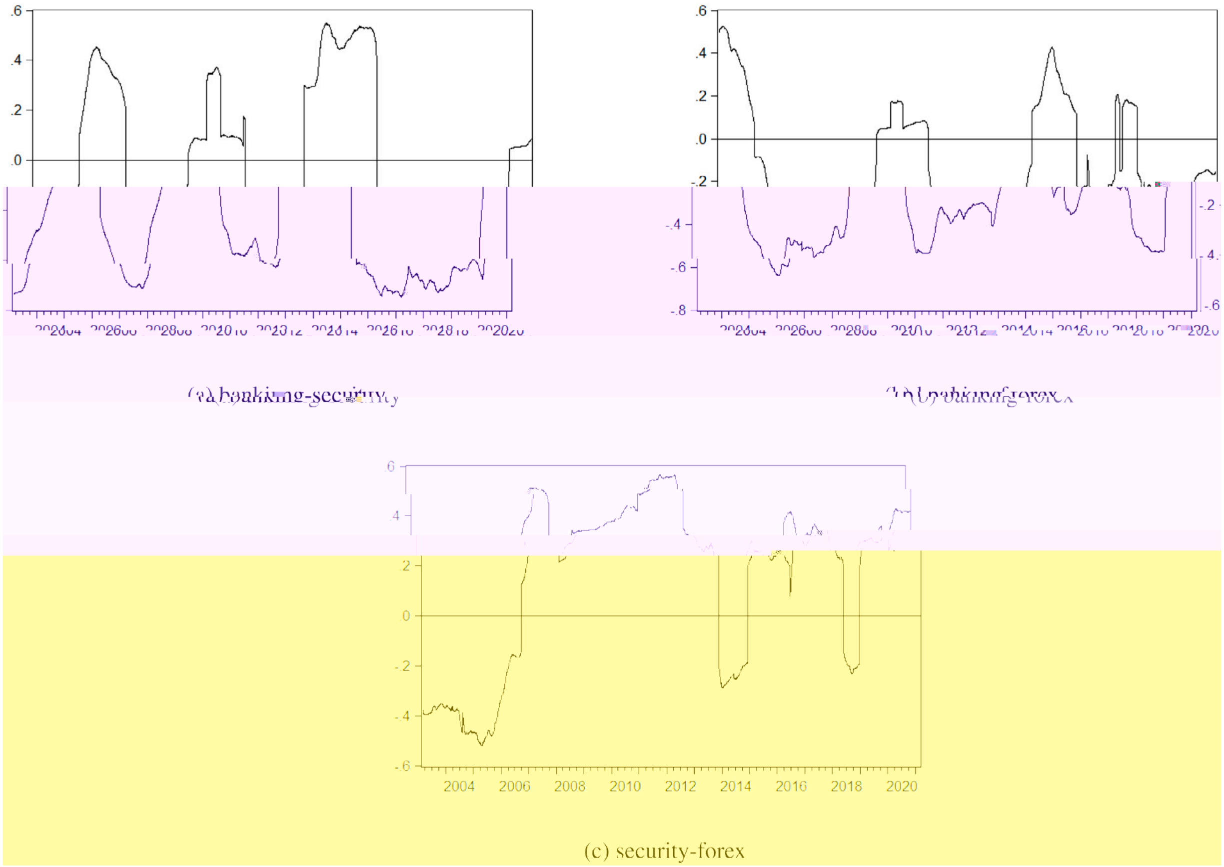

Fig. 3. Time-varying Kendall's Tau.

best fitted for the dependence structure of the banking–security pair, and the Clayton copula is the best fitted for the banking–forex and security–forex pairs. Kendall's Tau coefficients from the selected model are presented in Fig. 3.

6. Country-level FSSI

6.1. Aggregation of FSSI

The aggregation of individual stress indices is an important aspect of constructing the FSSI (Illing and Liu 2006). As shown in Section 3.3, we adopt a portfolio theory-based approach to building the FSSI. The first step of construction is to develop sub-component index for each sector (I_{BK}, I_{SC}, I_{FX}). The plots of three sub-component indices are presented in Fig. 1.

The next step is to aggregate the subindices based on the portfolio theory approach as stated in Eq. (1) using time-varying correlations C_t estimated by the best fitted copula model in Section 4. Fig. 4 provides an overview of the FSSI with selected major financial stress events.[5]

[5] The selection of major financial stress events is based on Craig (2020) and authors' consideration

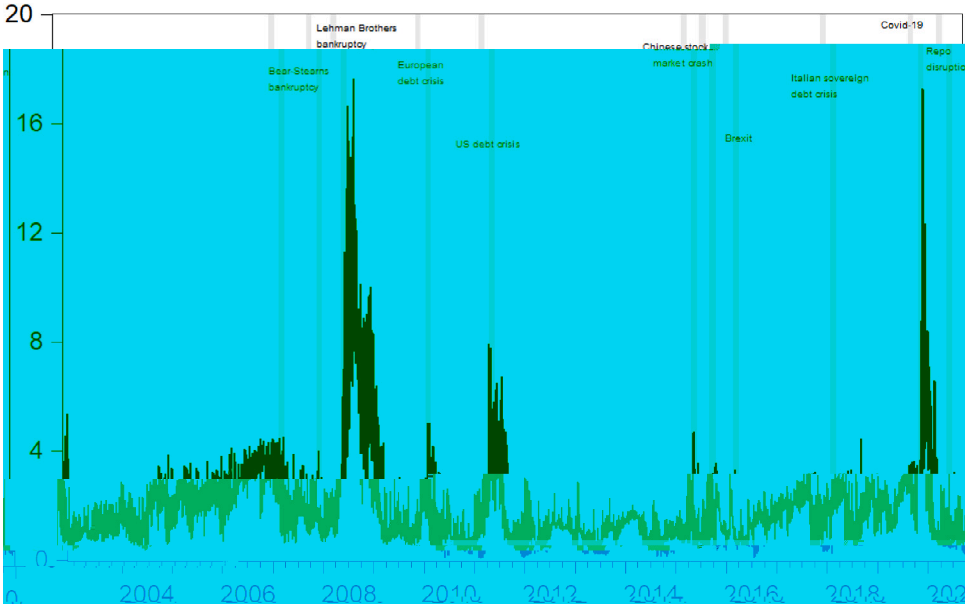

Fig. 4. Financial systemic stress index.

As seen from the figure, the systemic financial risk increases during the notably adverse financial events. The surge is most pronounced in the 2008 GFC, 2011 Black Monday stock crash and 2020 COVID-19 crisis periods. In March 2020 as COVID-19 pandemic spread globally and governments around the world shutdown nonessential business activities, mounting economic uncertainty led to a peak in financial stress, in which the intensity echoed that of 2008 GFC. However, the duration of systemic stress during the COVID-19 period is shorter than the GFC counterpart, reflecting swift rebounds in financial markets in tandem with the stronger-than-expected recovery of the global economy.

6.2. Identifying systemic financial risk

Using the FSSI constructed in the previous section, we can identify the states of systemic financial risk in the US financial system. In the existing literature, there are three commonly used methods to identify financial stress episodes. The approach of the Bank of Canada (see Illing and Liu (2006)) and International Monetary Fund is to classify financial stress as severe when the index exceeds the historical mean by one or two standard deviations. The disadvantage of this approach is that the choice of threshold is arbitrary, and the number of standard deviations by which the index exceeds the mean can change drastically with the presence of extreme observations. A second way of identification is to classify the FSSI as a risk episode whenever it equals or exceeds the value of the index in some benchmark crisis episodes, such as 1989 Black Monday or 2008 GFC. This approach is adopted in Hakkio et al. (2009). The final approach is to use the Markov-Switching (MS) model to identify different systemic financial risk states as in Hubrich and Tetlow (2015), Duprey and Klaus (2022), Duprey et al. (2017). In this paper, we follow the latter approach and build a Markov Switching Autoregressive (MS-AR) model to conduct the systemic risk episode identification.

MS model with fixed transition probability is proposed by Hamilton (1989). It provides a formal framework to investigate the presence of state switching by assuming that the data can be approximated by a mixture of two distributions with different mean and variance parameters. The MS-AR model employed here is ideal for several reasons. First, regime-switching of financial states is quite common due to financial cycles. The tranquil market indicates hidden low systemic risk regime, while the turbulent market suggests the high systemic risk regime and signals the possible crises. MS model is an ideal approach to find the hidden high stress regimes, which could be used to identify the crisis episodes. Second, the estimation of hidden regimes in MS model is only based on the data itself, which could avoid the arbitrary selection of threshold for crisis identification. Third, by incorporating lagged dependent variables into MS model, we can capture the autoregressive feature in financial markets and provide more accurate estimation for regime-switching.

To fit the FSSI data in the MS-AR model, we test the unit root of the series using the Augmented Dickey–Fuller test. The t-statistic of the test is $*4.753$ and the p-value is 0.0001, implying that FSSI is stationary at 1% significant level. To find the best two-regime MS-AR model, we estimate six models with number of lags from 1 to 6. The estimation results are shown in Table 5. According to the fitness criteria, MS-AR(6) is the best among six models. Hence, we adopt MS-AR(6) in the following identification. In the two-regime MS model, we set the low systemic risk state as 1 and high systemic risk state as 2. The MS-AR(6) model for the FSSI

Table 5
MS-AR models with different fitness criteria.

	Log likelihood	AIC	BIC	HQ
MS-AR(1)	*5901.280	2.514	2.522	2.517
MS-AR(2)	*5364.898	2.286	2.296	2.290
MS-AR(3)	*5881.618	2.507	2.518	2.511
MS-AR(4)	*5104.753	2.177	2.190	2.182
MS-AR(5)	*5033.547	2.148	2.162	2.153
MS-AR(6)	*5022.373*	2.144*	2.159*	2.149*

Notes: AIC, BIC and HQ represent Akaike information criterion, Bayesian information criterion and Hannan–Quinn information criterion respectively. * indicates the best model selected based on the highest log likelihood and the lowest AIC, BIC and HQ values.

Table 6
Transition matrix and state duration.

	Transition matrix		Expected duration	
	1	2	1	2
1	0.9863	0.0137	72.9055	1.5060
2	0.6640	0.3360		

can be expressed as:

$$FSSI_t = \begin{cases} c_1 + \sum_{i=1}^{6} \alpha_{1,i} FSSI_{t*i} + \varepsilon_{1,t} & \text{if } s_t = 1 \\ c_2 + \sum_{i=1}^{6} \alpha_{2,i} FSSI_{t*i} + \varepsilon_{2,t} & \text{if } s_t = 2 \end{cases} \tag{8}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are i.i.d and s_t is unobservable variable. The probability of switching between State 1 and State 2 follows the first order Markov chain. The transition matrix can be written as:

$$\mathbf{P} = \begin{bmatrix} P.s_t = 1|s_{t*1} = 1/ & P.s_t = 1|s_{t*1} = 2/ \\ P.s_t = 2|s_{t*1} = 1/ & P.s_t = 2|s_{t*1} = 2/ \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \tag{9}$$

The estimation results of transition matrix are shown in Table 6. $P_{11} = 0.9863$ means that the probability of the US financial stress staying at a low level is 0.9863. In other words, the low stress state is stable and hardly transits to a high stress state. Meanwhile, the probability of a high stress state is 0.3360 and it is relatively easy to transit to a low stress state with a probability of 0.6640. The expected duration for low stress and high stress is 72.9055 and 1.5060, respectively. This finding implies that systemic financial risk in the US is low for most periods of the sample, and high systemic risk only occurs for a few periods.

We further plot the regime probabilities in Fig. 5. Generally, the high stress periods are shorter than low stress periods. The high stress periods are clustering at 2008–2009, 2011 and 2020, which are exactly corresponding to the crisis periods highlighted in Fig. 4. This shows that the FSSI can be used as a crisis warning indicator. When the FSSI surges and the state is transiting to a high stress regime, the probability of systemic financial crisis increases. Take 2008 GFC as an example: the market state transited from low to high in early 2008, indicating the increase of systemic financial risk in the US. As the financial stress index constructed in this paper is in daily frequency, the index is more volatile during high stress periods. The state transition is also more frequent, which can be considered as another signal of excessive systemic financial risk.

7. Robustness tests

In this section, we carry out robustness checks by (1) using Principal Component Analysis (PCA) as an alternative approach to combine the market-specific variables, (2) constructing FSSI for other countries (Japan and Canada), and (3) considering additional financial indicators for inclusion in the FSSI.

7.1. PCA method for market-specific stress

In Section 3, when constructing the stress index for each market, we simply standardize each indicator and take the average of all indicators for that market. By adopting this simple average method, we assume that each indicator contributes equally to the market stress. The equal-weighting method for constructing subindices has been widely used in existing literature, such as (Cardarelli et al., 2011) and Hollo et al. (2012), given its simplicity in implementation and interpretation. Another popular approach to individual indicator aggregation is PCA, which involves identifying a common component among several variables (See Hakkio et al.,2009 and Oet et al., 2015). To test the robustness of baseline market-specific stress, we adopt the PCA method to reconstruct the banking market stress and security market stress. As shown in Fig. 6, the PCA-based subindices are very close to the baseline subindices.

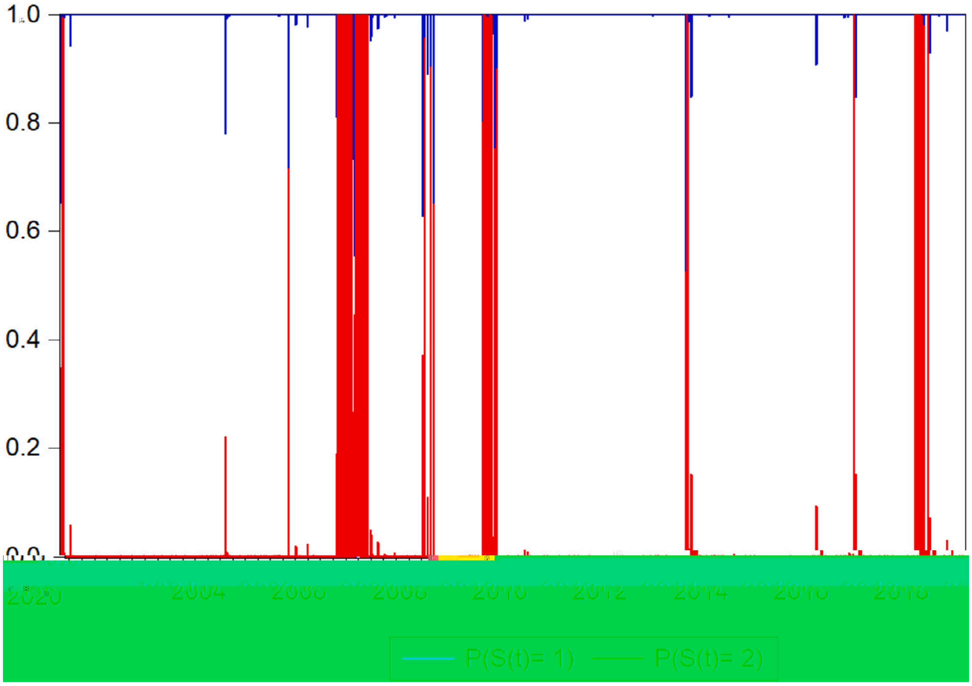

Fig. 5. Markov switching smoothed regime probabilities.

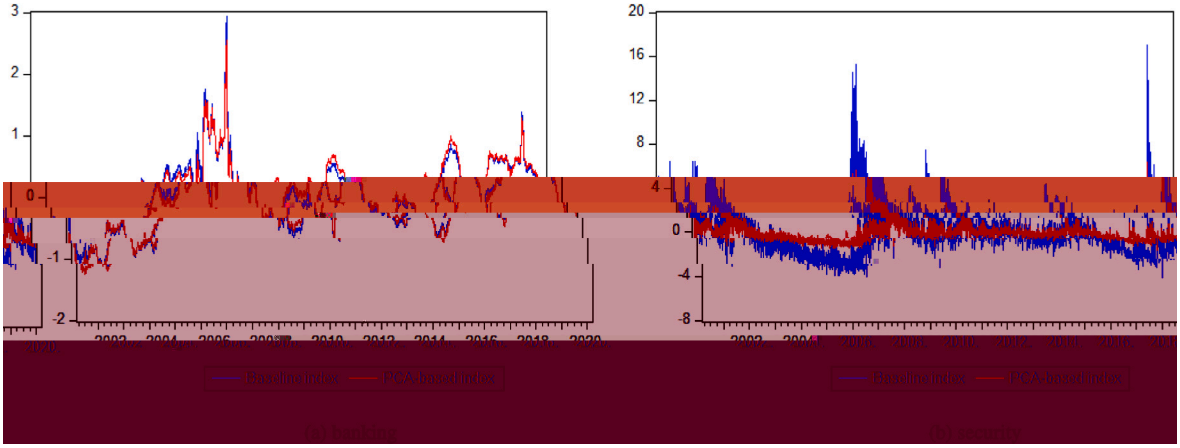

Fig. 6. Market-specific stress subindices with different constructing methods.

For the banking market index, the PCA-based index almost overlaps with the baseline index. For the security market index, the PCA-based index is less volatile but with similar upswings as those of the baseline index. As the forex market includes only one indicator, we cannot conduct the comparison analysis in terms of constructing method.

7.2. Other countries' FSSI

As the second robustness check, we apply the copula-based method to construct the FSSI for another two countries, which are Japan and Canada. The dependence structure among the market-specific stresses are modeled by the best time-varying copula selected based on the average AIC as presented in Tables 7 and 8 for Japan and Canada, respectively.[6] For Japan, the Joe copula is the best fitted for the dependence structure of the banking–security pair, and the Clayton copula is the best fitted for the banking–forex

[6] We also apply the static copula to model the dependence structure among the market-specific stresses. Detailed results are available upon request.

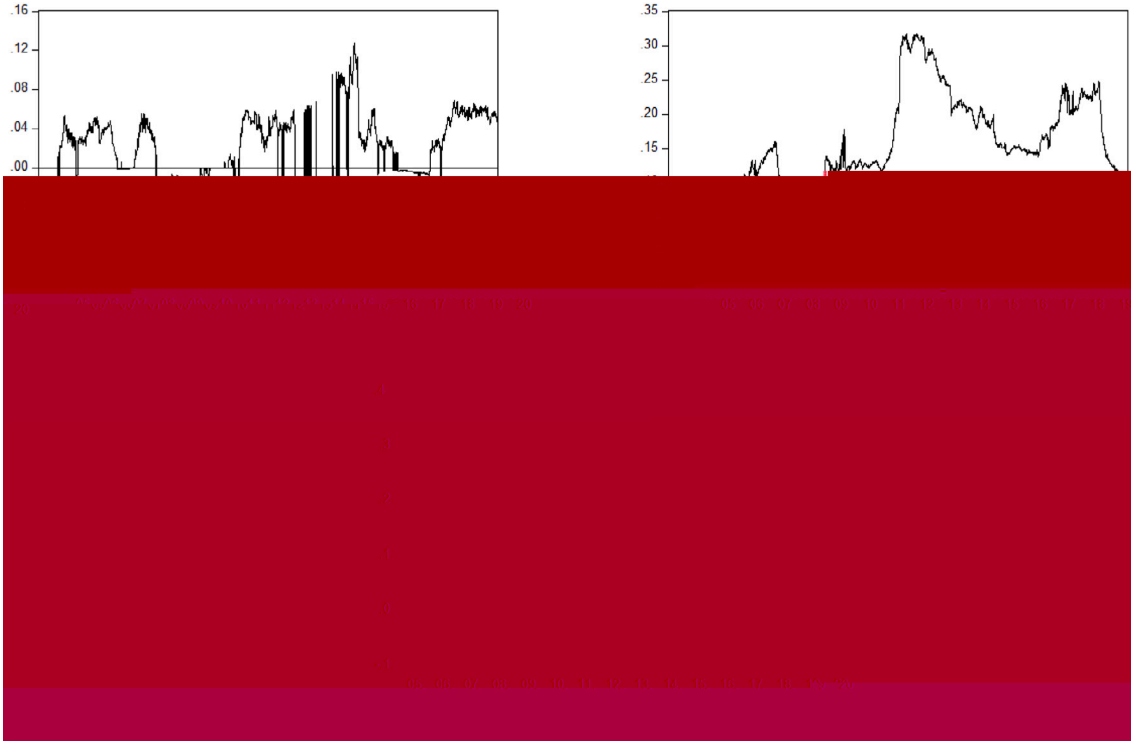

Fig. 7. Time-varying Kendall's Tau (Japan).

Table 7
Average AIC for time-varying copula (Japan).

Copula	Banking–security	Banking–forex	Security–forex
Gaussian	*100.2921	1.2701	*42.2413
Student t	*100.2613	3.6917	*41.1273
Clayton	*132.4662	0.3907	*36.5903
Gumbel	*116.5663	0.6742	*39.6243
Frank	*111.2390	0.9666	*38.0559
Joe	*132.6816	0.3929	*30.8667

Notes: The bold value is the best fitted copula model based on the lowest AIC.

Table 8
Average AIC for time-varying copula (Canada).

Copula	Banking–security	Banking–forex	Security–forex
Gaussian	*30.4605	0.7042	0.2981
Student t	*34.8469	2.6923	1.8918
Clayton	*49.9541	*0.0189	*0.5137
Gumbel	*43.0594	*0.0716	*0.6641
Frank	*33.9212	0.7034	0.6009
Joe	*50.2250	*0.1817	*0.8963

Notes: The bold value is the best fitted copula model based on the lowest AIC.

pair. Only the security–forex pair for Japan is best fitted by the Gaussian copula. On the other hand, all the dependence structure among the market-specific stresses of Canada is best fitted by the Clayton copula. Our results suggest that copula method is needed to model the dependence structure of stress subindices, as they are best modeled by Archimedean copula, implicating heavy tails occur among the market-specific stresses and normal distribution is not an appropriate assumption to model their correlations. The Kendall's Tau coefficients estimated from the selected copula model for all markets are presented in Figs. 7 and 8 for Japan and Canada, respectively. Based on the market-specific stress indices and market dependence coefficients estimated from the best fitted copulas, we further construct the FSSIs for Japan and Canada, as shown in Figs. 9 and 10. The upsurges in our FSSIs reflect the high financial stress episodes in the country's financial system as highlighted in the figures.

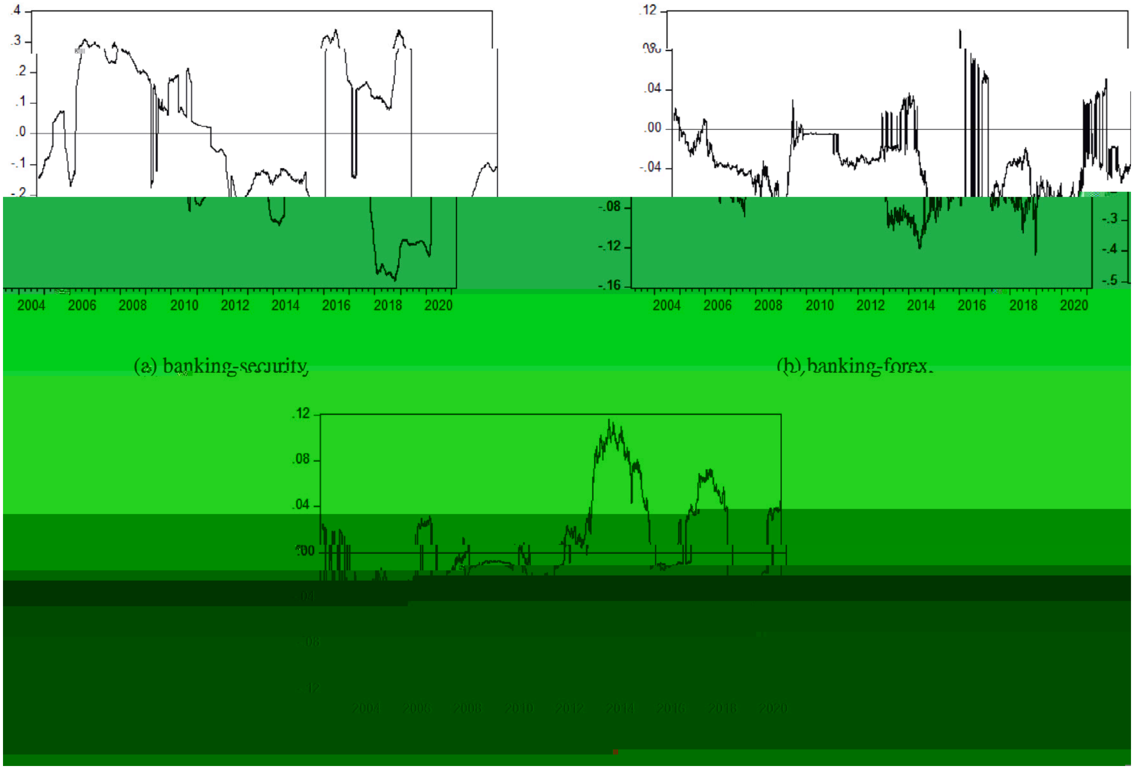

Fig. 8. Time-varying Kendall's Tau (Canada).

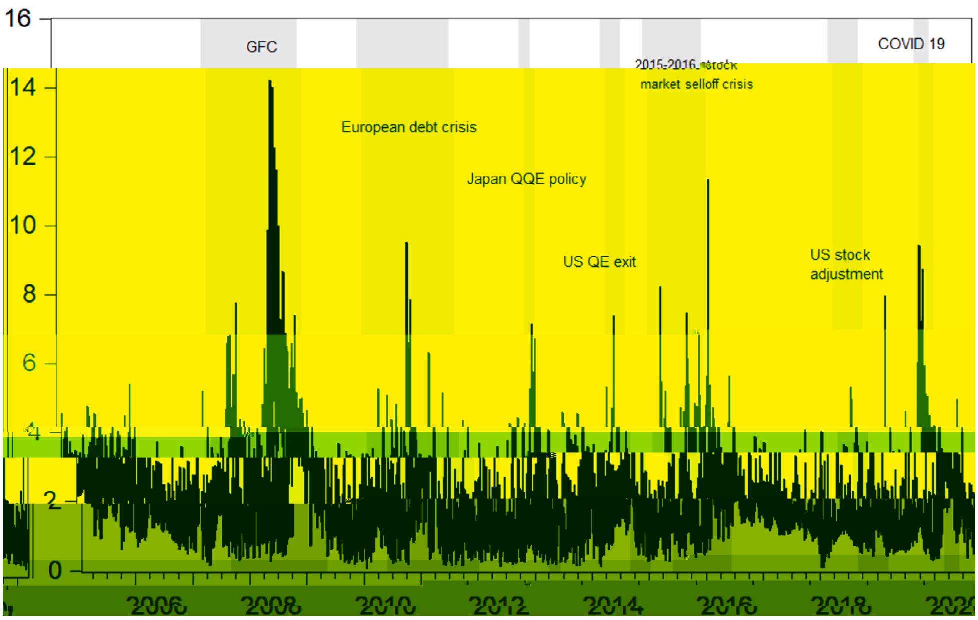

Fig. 9. FSSI for Japan.

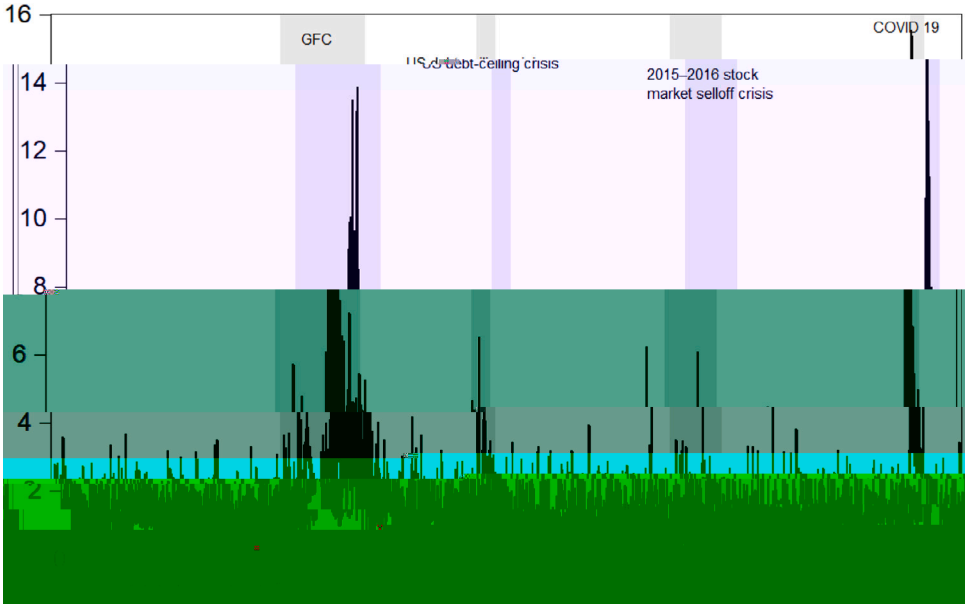

Fig. 10. FSSI for Canada.

Table 9
Average AIC for time-varying copula (US).

Copula	Banking–security	Banking–forex	Security–forex
Gaussian	*190.3598	*85.5131	*177.2476
Student t	*198.5411	*88.8921	*180.7726
Clayton	*246.7964	*126.3831	*203.5989
Gumbel	*216.8472	*105.0465	*192.0354
Frank	*189.5934	*82.7213	*157.8610
Joe	*246.5431	*127.5320	*200.3100

Notes: The bold value is the best fitted copula model based on the lowest AIC.

7.3. Additional indicators in the FSSI

Finally, we conduct a robustness check by considering additional financial indicators à la (Hollo et al., 2012) to our baseline FSSI. The additional variable included in the banking market stress is described below:

- Realized volatility of the 3-month US's LIBOR rate: realized volatility calculated as the weekly average of absolute daily rate changes.

Also, two variables are added to the security market stress, as listed below:

- Realized volatility of the US 10-year government bond yield: realized volatility calculated as the weekly average of absolute daily yield changes.
- 10-year interest rate swap spread: swap spread calculated as the difference between 10-year swap rate and 10-year government bond yield.

The dependence structure among the market-specific stresses of US are modeled by the best time-varying copula which is selected based on the average AIC as presented in Table 9.[7] The Clayton copula is the best fitted for the dependence structure of the banking–security pair and equity–forex pair, and the Joe copula is the best fitted for the banking–forex pair. The results are consistent with the baseline results as reported in Table 4. The Kendall's Tau coefficients from the selected model are presented in Fig. 11. We further construct the US's FSSI using these market-specific stresses and copula-based dependence coefficients. This new FSSI constructed with additional variables is compared with the baseline FSSI in Fig. 12. The fluctuations of FSSI with additional variables are largely consistent with the baseline FSSI.

[7] We also apply the static copula to model the dependence structure among the market-specific stresses. Detailed results are available upon request.

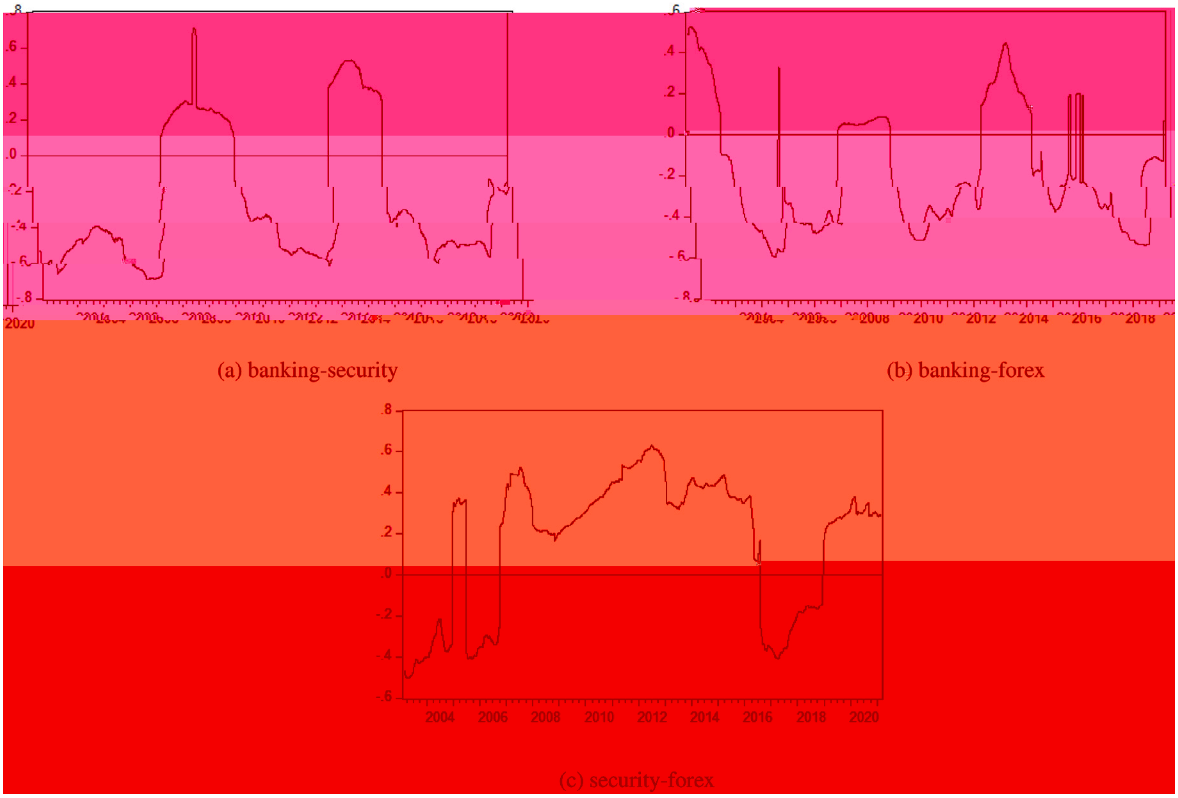

Fig. 11. Time-varying Kendall's Tau (US).

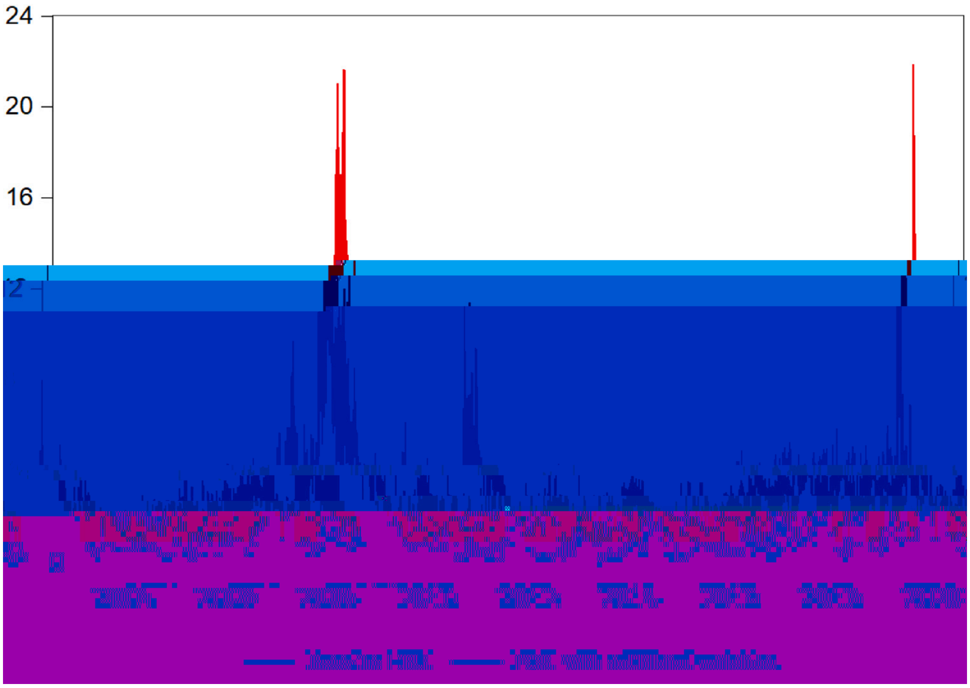

Fig. 12. Baseline FSSI vs FSSI with additional variables.

8. Conclusion

This work proposes a reproducible approach to measuring and monitoring financial stress, thus having important implications for the macroprudential regulation of the financial system. This is especially pertinent given the surge in financial frailty during the ongoing COVID-19 crisis. We combined the portfolio-theory based approach proposed by Hollo et al. (2012) and the copula method, which offers a flexible method for modeling nonlinear dependence structures across financial sectors. This way, our composite indicator for financial stress can capture the systemic nature of a crisis, which is not adequately represented by the linear correlation measures, and reduce the chance of underestimating financial risk in the presence of asymmetric dependence between financial segments.

We find that the dependence structures between the market-specific stresses are best explained by Archimedian copula families. Specifically, in the dynamic setting, the Joe and Clayton copulas are the best models based on AIC, signaling the presence of asymmetric tail dependencies between individual market stresses over time.

Using Kendall's Tau coefficients from the best fitted time-varying copulas, the market specific stresses are aggregated to a single FSSI. We apply our MS-AR model to identify different states of systemic financial risk based on the FSSI. We find that most of the sample periods constitute low stress episodes and a few periods can be classified as high stress states. The Markov state identification result, based on FSSI data, is able to detect several recent instances of financial turbulence in the US and confirms that the FSSI is a reliable indicator for measuring systemic financial risk.

Our construction method for the country-specific FSSI can be considered the first step towards a more in-depth analysis of financial instability. For instance, the copula method can be used to measure the systemic stress experienced by regional and global financial systems. We may also apply the FSSIs of a wide range of markets to study various important topics in international finance, such as financial stress spillovers across regions, and linkages between the FSSIs and other economic and financial indicators.

CRediT authorship contribution statement

Sook-Rei Tan: Conceptualization, Methodology, Project administration, Funding acquisition, Writing – review & editing. Changtai Li: Visualization, Formal analysis, Investigation, Software, Writing – original draft. Xiu Wei Yeap: Data curation, Methodology, Formal analysis, Software, Writing – original draft.

Data availability

Data will be made available on request.

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