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E al a ing he speci ca ion errors of asse pricing models ☆

Rober J. Hodrick[a,b,*], Xiao an Zhang[a]

[a] *G oo o B , Co v , H 605 , 3022 Bo , N o , N 10027, A*

[b] *N o B o Eo o , 1050 M Av , C g , MA 02138, A*

A

This paper e al a es he speci ca ion errors of se eral empirical asse pricing models ha ha e been de eloped as po en ial impro emen s on he CAPM. We se he me hodolog of Hansen and Jaganna han (J. Finance 51 (1997) 3), and he es asse s are he 25 Fama-French (J. Financial Econom. 52 (1997) 557) eq i por folios sor ed on si e and book- o-marke ra io, and he Treas r bill. We allo he parame ers of each model's pricing kernel o c a e i h he b siness c cle. While e canno rejec correc pricing for Campbell's (J. Poli ical Econom. 104 (1996) 298) model, s abili es s indica e ha he parame ers ma no be s able. A rob s ness es also indica es ha none of he models correc l price re rns ha are scaled b he erm premi m. © 2001 Else ier Science S.A. All righ s reser ed.

JEL o : C52; G11; G12

K o : Hansen-Jaganna han dis ance; Asse pricing; Time- ar ing risk prices

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*Corresponding a hor. Grad a e School of B siness, Col mbia Uni ersi , Uris Hall 605b, 3022 Broad a , Ne York, NY 10027, USA. Tel. +1-212-854-3413; fa +1-212-316-9219.

E- : rh169@col mbia.ed (R.J. Hodrick).

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1.

Thro gho he 1970s and 1980s, nancial economis s in es iga ed he pricing implica ions of he capi al asse pricing model (CAPM) de eloped b Sharpe (1964) and Lin ner (1965). The ell-kno n predic ion of he CAPM is ha he e pec ed e cess re rn on an asse eq als he co ariance of he re rn on he asse i h he re rn on he marke por folio imes he marke price of risk. This price is he ra io of he e pec ed e cess re rn on he marke por folio o he ariance of he re rn on he marke por folio. The e pec ed re rn predic ion of he CAPM can eq i alen l be s a ed as he be a of he asse imes he e pec ed e cess re rn on he marke por folio, here he be a is he co ariance of he asse 's re rn i h he re rn on he marke por folio di ided b he ariance of he marke re rn.

As empirical research began o nco er a n mber of e pec ed-re rn anomalies ha he CAPM co ld no e plain, Roll (1977) arg ed ha he model as no es able. Beca se in es ors and rms assessing heir cos s of capi al an o kno he de erminan s of e pec ed re rns, empirical research con in ed, b i as necessaril cond c ed nder he recogni ion ha he es s in ol e a join h po hesis on he model and he choice of he marke por folio.

The inabili of he CAPM o e plain he cross-sec ion of asse re rns led o he de elopmen of a n mber of al erna i e empirical asse pricing models. The di ersi of hese models and he fac ha he ha e been e al a ed on a arie of da a se s pose se ere di c l ies for someone ho is r ing o nders and if an of hese models is a reasonable replacemen for he CAPM. The p rpose of his paper is o e al a e and compare a n mber of hese models on a common da a se sing an appropria e me hodolog .

Par of o r empirical anal sis ses he me hodolog of Hansen and Jaganna han (1997), ho de elop a dis ance me ric e call he HJ-dis ance. Hansen and Jaganna han demons ra e ho o meas re he dis ance be een a r e pricing kernel (s ochas ic disco n fac or) ha prices all asse s, and he implied pricing kernel pro of an asse pricing model. The dis ance be een hese o random ariables is calc la ed in he s al a as he sq are roo of he e pec ed al e of he sq ared di erence be een he o ariables. HJ-dis ance can also be in erpre ed as he normali ed ma im m pricing error of he model for por folios formed from ha se of asse s. Th s, if he model is correc , he HJ-dis ance is ero, and here are no pricing errors. Glasserman and Jin (1998) pro ide an al erna i e a of comparing models of s ochas ic disco n fac ors (SDF) b e amining he ph sical probabili meas res of asse he Sharpe ra io predic ed b he model and he r e Sharpe ra io. Conseq en l , es ima ion of HJ-dis ance also pro ides he ma im m e pec ed re rn error of he model b ass ming he in es or ses a par ic lar s andard de ia ion.

The models e e amine o from he de elopmen of he li era re. E en before he CAPM anomalies began o acc m la e, heoris s s ch as Mer on (1973) no ed ha he CAPM is a s a ic model, and he de eloped in er emporal models in hich co ariances of re rns i h s a e ariables o her han he marke re rn co ld in ence e pec ed re rns if he cons mp ion and in es men oppor ni se s of in es ors ar o er ime. Breeden (1979) de eloped a Cons mp ion CAPM (CCAPM) b demons ra ing ha an asse 's risk premi m depends on he co ariance of he asse 's re rn i h aggrega e cons mp ion in con in o s ime d namic op imi a ion models. Hansen and Single on (1982) de eloped an empirical es of he CCAPM in discre e ime b sing he E ler eq a ion of he in es or's d namic op imi a ion problem, in hich an e pec ed re rn depends on he co ariance of he re rn i h he marginal ili of cons mp ion.

The empirical fail re of he CCAPM and he heore ical appeal of he Mer on logic led Campbell (1993, 1996) o de elop a d namic asse pricing model in hich an e pec ed re rn depends on he co ariances of he re rn i h he marke por folio and i h he inno a ion in he presen disco n ed al e of f re e pec ed marke re rns. In he Campbell model, an hing ha forecas s marke re rns becomes a risk fac or for asse re rns.

Jaganna han and Wang (1996) no ed ha i is possible for he CAPM o hold as a condi ional model of e pec ed re rns i h condi ional be as, b he ncondi ional model o ld be more complica ed since be as co ld ar o er ime. The de eloped an empirical model of his be a-premi m sensi i i b aking a s and on he na re of he predic abili of marke re rns.

Cochrane (1996) responded o he fail re of he CCAPM b no ing ha he prod c ion side of he econom also m s sa isf d namic E ler eq a ions. This logic led him o de elop he implica ions of a prod c ion-based asse pricing model in hich co ariances of asse re rns i h macroeconomic meas res of in es men are impor an risk fac ors.

Finall , he empirical fail re of he CAPM and he heore ical appeal of m l i-fac or models led Fama and French (1992, 1993, 1995, 1996) o de elop a hree-fac or model. I is fair o sa ha his ne model, or some e ended arian of i , is no he orkhorse for risk adj s men in academic circles.

Al ho gh he es ima ion of he parame ers associa ed i h he meas remen of HJ-dis ance sol es a generali ed me hod of momen s (GMM) problem ha minimi es a q adra ic form based on he a erage pricing errors from he basic asse s, i is no he op imal GMM of Hansen (1982). We also repor res l s from op imal GMM es s of he models, and e generall nd similar inference abo he alidi of he models as in he HJ-dis ance problems. Nei her of hese approaches direc l minimi es he pricing errors of he basic asse s hich is eq i alen o sing an iden i ma ri in GMM es ima ion. While s ch es ima ion is pop lar and sa is es he e es' desire for small errors, inference abo he alidi of he models is a ec ed se erel b he increase in he s andard errors associa ed i h his approach. Conseq en l , e do no repor hese res l s.

Beca se here is considerable e idence ha e pec ed re rns c a e o er ime, e an o allo for ime- ar ing prices of risks. We do his b allo ing he parame ers of he models o c a e i h he b siness c cle. We meas re he b siness c cle in o a s. One ses he Hodrick and Presco (1997) l er applied o ei her ind s rial prod c ion for mon hl models or real GNP for q ar erl models. The second approach for q ar erl models ses he cons mp ion– eal h meas re de eloped b Le a and L d igson (2001a, b). Also, beca se Lo ghran (1997) and Daniel and Ti man (1997) arg e ha re rn charac eris ics are di eren in Jan ar han o side of Jan ar , e se a Jan ar d mm ariable o allo he parame ers of he models o di er across his mon h and he o her mon hs.

Bo h HJ-dis ance and op imal GMM ass me ha he parame ers of he model are s able o er ime. If a model is misspeci eqj aZPl beca se i s parame ers no s able, i ma ne er heless pass he es of HJ-dis ance eq als ero, b i o ld no predic ell o -of-sample. This si a ion can charac eri e bo h condi ional and ncondi ional models. Gh sels (1998) nds ha sing condi ioning ariables o impro e asse pricing models ma ac all orsen heir performance o -of-sample beca se of parame er ins abili . We herefore follo Gh sels ho ses he s pLM es de eloped b Andre s (1993) o in es iga e ins abili in parame ers.

The common re rns ha e req ire each of he models o price are he re rns on he 25 por folios cons r c ed b Fama and French (1993) in hich rms are sor ed b he marke al e of heir eq i (si e) and he book- o-marke ra io. We se re rns in e cess of he Treas r bill re rn, and e also req ire he models o price he Treas qj aZq r ZP-j M- billZP-4qM4 re rZP Mj n.ZP-4ql he econome ric aspec s of he paper incl ding he deri a ions of HJ-dis ance, he es ha HJ-dis eq als ero, and he in erpre a ion of HJ-dis ance as he ma im m di erence be een he Sharpe ra io of he model and he r e Sharpe ra io. Sec ion 3 disc sses he da a and he parame eri a ion of he di eren models. Sec ion 4 con ains he empirical res l s. Sec ion 5 pro ides concl ding remarks.

2.

2.1. Mo

Ass me e ha e basic asse s o be priced. I is ell kno n ha in he absence of arbi rage oppor ni ies here e is s a se M of s ochas ic pricing kernels hich price e er asse correc l . Tha is,

$$\mathrm{E}\,(\quad _{+1} \quad _{,+1}) = \quad , \quad \forall \;, \; > 0, \; \forall \quad _{+1} \in M_{+1}, \tag{1}$$

here $_{+1}$ is he s ochas ic pricing kernel a ime $+1$, M_{+1} is he se of correc pricing kernels, $_{,+1}$ is he re rn for por folio a ime $+1$, and he price for re rn $_{,+1}$ a ime is . If $_{,+1}$ is a gross re rn for a por folio, hen $= 1$; if $_{,+1}$ is an e cess re rn for a por folio, hen $= 0$. The condi ional e pec a ion in Eq. (1) is based on he informa ion se a , deno ed Φ . B he la of i era ed e pec a ions, he ncondi ional ersion of Eq. (1) is

$$\mathrm{E}(\quad _{+1} \quad _{,+1}) = \quad , \quad \forall \;, \; > 0, \; \forall \quad _{+1} \in M_{+1}. \tag{2}$$

We se Eq. (2) o es ima e and es he ario s asse -pricing models.

As Hansen and Jaganna han (1997) no e, an asse pricing model pro ides a pricing kernel pro , $_{+1}$. If he model is r e, $_{+1} \in M_{+1}$. We ill e amine models in hich he pricing pro is a linear f nc ion of a cons an and a ec or of ariable fac ors, $_{+1}$. De ne $F'_{+1} = [1, \; '_{+1}]$, and le he ec or of parame ers be $' = [\; _0, \; '_1]$. Then he pricing pro is

$$_{+1} = \; ' F_{+1} = \; _0 + \; '_1 \; _{+1}, \tag{3}$$

here F_{+1} is he $\times 1$ fac or ec or, and is he $\times 1$ coe cien ec or. Non ero elemen s of indica e he impor ance of a fac or as a de erminan of he pricing kernel. For ease of presen a ion, e drop he ime s bscrip hen i is no necessar for clari of presen a ion.

Cochrane (1996) no es ha if he model is r e, Eq. (2) holds for all asse s i h $_{+1}$ s bs i ed for $_{+1}$. Then, if is he $\times 1$ ec or of 's, he pricing model has an eq i alen represen a ion in erms of m l i aria e be as and prices of risks

$$\mathrm{E}(\;) = \; ^0 \; + \beta' \Lambda, \tag{4}$$

here $^0 = 1/E(\;)$, $\beta = \; o\,v(\;, \; ')^{-1} \; o\,v(\;, \; ')$, and $\Lambda = - \; ^0 \; o\,v(\;, \; ') \; _1$.

In Eq. (4), ^{0} is he ncondi ional risk-free ra e or he ero-be a ra e, he β's de ermine he her he h fac or signi can l in ences he e pec ed re rns on a par ic lar se of por folios, e m s assess he her he corresponding Λ is signi can l di eren from ero. No ice $\Lambda = 0$ does no mean $_{1,} = 0$ and ice ersa. Onl hen $o\,v(\;, \;')$ is diagonal are he o s a emen s eq i alen . The deri a ions and proofs of hese s a emen s can be fo nd in Cochrane (1996).

One m s be clear in disc ssing he prices of fac or risks he her i is be a risk or co ariance risk. Campbell (1996), for e ample, ses he co ariance decomposi ion of Eq. (2) o ri e

$$\mathrm{E}(\;) = \;^{0} - \;^{0}\, o\,v(\;, \;). \tag{5}$$

B s bs i ing he de ni ion of $_{+1}$ for $_{+1}$ in Eq. (5), one can ri e

$$\mathrm{E}(\;) = \;^{0} + \sum_{=1} \; o\,v(\;, \;), \tag{6}$$

here he price of he h co ariance risk is $= - \;^{0}\;_{1,}$. Since ^{0} is no er di eren from one, e do no repor s a is ics for .

2.2. HJ-

Hansen and Jaganna han (1997) no e ha hen he asse pricing model is false, $\notin M$, and here is a s ric l posi i e dis ance be een and M. Hansen and Jaganna han de ne he dis ance, hich e call HJ-dis ance, as

$$\delta = \min_{\in L^2} \| \; - \; \|, \quad \text{here } \mathrm{E}(\quad) = \;, \tag{7}$$

and he meas re of dis ance is he s al norm, $\| \; \| = \sqrt{\mathrm{E}(\;^2)}$.[1] The problem de ned in Eq. (7) can be re ri en as he follo ing Lagrangian minimi a ion problem

$$\delta^2 = \min_{\in L^2} \; \mathrm{s\;p}_{\lambda \in} \{\mathrm{E}(\; - \;)^2 + 2\lambda'[\mathrm{E}(\quad) - \;]\}. \tag{8}$$

The al e of δ is he minim m dis ance from he pricing pro o he se of r e pricing kernels M. Le $\tilde{}$ and $\tilde{\lambda}$ be he sol ion o Eq. (8). One can hink of $- \;\tilde{}$ as he minimal adj s men o o make i a r e pricing kernel. Hansen and Jaganna han (1997) sol e Eq. (8) o nd

$$- \;\tilde{} = \tilde{\lambda}' \;, \tag{9}$$

here

$$\tilde{\lambda} = \mathrm{E}(\quad ')^{-1}\mathrm{E}(\quad - \;). \tag{10}$$

[1] Hansen and Jaganna han (1997) also consider a dis ance meas re in hich is req ired o be s ric l posi i e. If he problem is sol ed i ho he cons rain and $_{+1} > 0$ for all , he o sol ions coincide. In heir empirical anal sis, Hansen and Jaganna han nd his addi ional res ric ion does no make a big di erence.

Th s, he HJ-dis ance is

$$\delta = \| \quad - \tilde{\ } \| = \|\tilde{\lambda}' \quad \| = [\tilde{\lambda}'\mathrm{E}(\quad ')\tilde{\lambda}]^{1/2}. \tag{11}$$

S bs i ing for he al e of $\tilde{\lambda}$ from Eq. (10) gi es

$$\delta = [\mathrm{E}(\quad - \quad)'\mathrm{E}(\quad ')^{-1}\mathrm{E}(\quad - \quad)]^{1/2}. \tag{12}$$

B sol ing he conj ga e problem o Eq. (8), Hansen and Jaganna han (1997) also pro ide an impor an al erna i e in erpre a ion o δ. I is he ma im m pricing error for he se of por folios based on he basic asse pa o s i h he norm of he por folio re rn eq al o one. We follo Campbell and Cochrane (2000) in in erpre ing he re rn errors of he models sing his logic.

Consider he re rn on a por folio of he basic asse s, θ' . The r e e pec ed re rn for his por folio hen priced i h $\tilde{\ }$ is fo nd from Eq. (5) o be

$$\mathrm{E}(\theta' \quad) = \quad {}^{0}\theta' \quad - \quad {}^{0} \; o\,v(\tilde{\ }, \theta' \quad). \tag{13}$$

Le E $(\theta' \quad)$ deno e he e pec ed al e of he por folio re rn predic ed b he pricing pro . When E( ) = E($\tilde{\ }$) = ($^{0})^{-1}$, e can ri e

$$\mathrm{E} \; (\theta' \quad) = \quad {}^{0}\theta' \quad - \quad {}^{0} \; o\,v(\quad , \theta' \quad). \tag{14}$$

B s b rac ing Eq. (14) from Eq. (13) and sing he Ca ch -Sch ar ineq ali , e ha e

$$|\mathrm{E}(\theta' \quad) - \mathrm{E} \; (\theta' \quad)| = | \quad {}^{0} \; o\,v(\quad - \tilde{\ }, \theta' \quad)| \leqslant \quad {}^{0}\sigma(\quad - \tilde{\ })\sigma(\theta' \quad), \tag{15}$$

here $\sigma(\;)$ deno es he s andard de ia ion of . The ineq ali in Eq. (15) holds as an eq ali hen he por folio re rn is perfec l correla ed i h $- \tilde{\ }$. Recall from Eq. (9) ha $\tilde{\lambda}' \quad = \quad - \tilde{\ }$, and $\delta = \sigma(\quad - \tilde{\ })$ hen E( ) = E($\tilde{\ }$). Th s, he por folio i h shares $\theta = \tilde{\lambda}/\delta$ is he ma imall mispriced por folio i h norm eq al o one. S bs i ing hese res l s in o Eq. (15) and recogni ing ha $\mathrm{E}(\lambda' \quad) = 0$ gi es

$$\frac{|\mathrm{E} \; (\lambda' \quad)|}{\sigma(\lambda' \quad)} = \quad {}^{0}\delta. \tag{16}$$

The lef -hand side of Eq. (16) is he ma im m absol e pricing error per ni of s andard de ia ion, or he ma im m mispriced Sharpe ra io. Campbell and Cochrane (2000) e ploi his idea o e al a e ann ali ed e pec ed re rn errors of false models b m l ipl ing $^{0}\delta$ b an ann ali ed s andard de ia ion of 20%. We repor his pe of model re rn error belo .

2.3. *E o o*

Hansen and Jaganna han (1997) no e ha $\hat{\ }$, he es ima e of , can be chosen o minimi e δ. To see he rela ion of his problem o a s andard generali ed me hod of momen s (GMM) problem, de ne he pricing error ec or $g = \mathrm{E}(\quad - \quad)$, and i s sample co n erpar

$$g \ (\ ) = \frac{1}{} \sum_{=1} \quad - \quad , \tag{17}$$

and le be a sample es ima e of $\mathrm{E}(\quad ')^{-1}$. Then, b sq aring Eq. (12), $\hat{}$ can be chosen as

$$\hat{} = \arg\min \delta^2 = \arg\min g' \ (\ ) \quad {}_g \ (\ ). \tag{18}$$

While Eq. (18) is a s andard GMM problem, i is no he op imal GMM of Hansen (1982) hich ses as he eigh ing ma ri , $^* = \ ^{-1}$, here is a consis en es ima or of $^* \equiv [\quad v \quad (g \ )]$. Hansen demons ra es ha * is op imal in he sense ha he es ima ed parame ers ha e he smalles as mp o ic co ariance.

In general, he op imal eigh ing ma ri assigns big eigh s o asse s i h small ariances in heir pricing errors, and i assigns small eigh s o asse s i h large ariances of heir pricing errors. I is ob io s ha * changes i h di eren models. This makes i ns i able for he ask of making comparisons among compe ing models. The al erna i e eigh ing ma ri of Hansen and Jaganna han (1997) is in arian across compe ing asse pricing models. Using a common eigh ing ma ri allo s s o ha e a niform meas re of performance across models for a common se of por folios. The onl ass mp ion needed is ha he eigh ing ma ri is nonsing lar.

Cochrane (1996) arg es ha $\mathrm{E}(\quad ')$ ma be nearl sing lar in hich case he in ersion is problema ic, b as e disc ss la er, e did no enco n er in ersion problems. To a oid in ersion problems and o keep he eigh ing ma ri he same across asse s, Cochrane ses he iden i ma ri as a eigh ing ma ri . This approach is of en done in he rs -s age es ima es of a GMM problem beca se es ima ion of * req ires consis en es ima es of he parame ers.

B assigning eq al eigh s o all basic asse s and ignoring cross prod c s of pricing errors, Cochrane's (1996) approach minimi es he s m of sq ared pricing errors, hich is appealing for o reasons. Firs , i is eq i alen o a radi ional leas sq ares approach of en sed in nance, and second, i pro ides he bes graphical represen a ion of predic ed re rns on he basic asse s ers s heir a erage re rns.

These desirable a rib es m s be balanced agains he heore ical appeal of ei her op imal GMM or he HJ-dis ance approach. Op imal GMM pro ides he mos e cien es ima es among es ima es ha se linear combina ions of pricing errors as momen s. Working i h he smalles s andard errors pro ides a more po erf l a o es he alidi of a par ic lar model. B , beca se * is model dependen , i makes no sense o compare chi-sq are s a is ics across models.- We prefer he HJ-dis ance approach beca se i is e plici l designed for comparing he pricing errors of al erna i e models.-

Belo e repor s a is ics for bo h HJ-dis ance and op imal GMM.- We do no repor s a is ics from rs -s age es ima es beca se e fo nd hem rela i el ninforma i e.-Mos of he models ere no rejec ed j s d e o large s andard errors, hich is economicall nin eres ing.-We also do no nd big

Since v $[g$ $(\hat{})]$ onl has rank $-$, e se i s pse do in erse follo ing Cochrane (1996). For op imal GMM, his Wald es red ces o he ell-kno n J- es , i h

$$J = g'(\hat{})v \quad [g \ (\hat{})]^{-1} g \ (\hat{}) = \quad g \ (\hat{}) \quad {}^{*} g \ (\hat{})$$

$$\overset{\mathrm{d}}{\to} \chi^2(\ - \ ). \tag{25}$$

From Eq. (10) he co ariance ma ri of he Lagrange m l ipliers is

$$v \quad (\tilde{\lambda}) = \quad v \quad [g \ (\hat{})] \quad . \tag{26}$$

Since he ma im m pricing error δ is achie ed b θ' i h $\theta = \tilde{\lambda}/\delta$, e can e amine he impor ance of indi id al asse s o he pricing error b e amining he n ll h po hesis $\tilde{\lambda} = 0$.

Finall , i is impor an o dis ing ish hich pricing errors are nder disc ssion. We de ned he pricing errors of he models in Eq. (17). I is he sample a erage for he di erences in prices hen e se o price min s he correc prices hich sho ld be ero for an e cess re rn and one for a gross re rn. As in o her research, e can also de ne a erage re rn errors as

$$\pi = \bar{} - E \ (\ ) = \frac{1}{} \sum_{=1} \quad - \quad {}^{0}[\quad - o\,v(\ , \ )] = \quad {}^{0} g \ (\hat{}). \tag{27}$$

To a oid conf sion, e refer o g $(\hat{})$ as model errors and π as he pricing errors of he basic asse s. Since ^{0} di ers sligh l across models, he o do no pro ide he same informa ion. We look a g $(\hat{})$ mainl for de ails associa ed direc l i h δ. We e amine π o compare pricing errors for he basic asse s across models.

2.4. Co o o

E amining he ncondi ional implica ions of linear fac or models has o inheren problems. One is ha onl ncondi ional risk premi ms are es ima ed. The second is ha he models force prices of f ndamen al risks o be cons an across b siness c cles. Cochrane (1996), Ferson and Har e (1999), and o hers r o sol e hese o problems b sing macroeconomic ariables as condi ioning ariables. In Eq. (3), all parame ers in are cons an . To allo hem o ar i h some elemen in Φ , e ri e

$$
\begin{aligned}
{}_{+1} &= \ '(\ )F_{+1} \\
&= (\ {}_{0,1} + \ {}_{0,2} \ ) = [\ '_{1,1} + (\ {}_{1,2} \ )']F_{+1} \\
&\quad + \ {}_{0,1} + \ {}_{0,2} \ + \ '_{1,1}F_{+1} + \ '_{1,2}(F_{+1} \ ).
\end{aligned} \tag{28}
$$

The las eq al sign demons ra es Cochrane's poin , scaling he prices of fac ors is eq i alen o scaling he fac ors.

If prices of risks c a e o er he b siness c cle, e can cap re his e ec b sing ariables ha are associa ed i h b siness c cles. There are hree req iremen s for macroeconomic ariables o be legi ima e ins r men s. Firs , he m s be incl ded in he ime informa ion se . Second, he sho ld s mmari e he s a s of he b siness c cle. Third, since he n mber of he parame ers increases geome ricall i h he n mber of condi ioning ariables, hich can make he es ima es nreliable, he condi ioning ariables canno be oo n mero s. We se onl one condi ioning ariable a a ime. Beca se he pre io s li era re has foc sed on bo h mon hl and q ar erl hori ons, e o ld like a similar condi ioning ariable for each hori on.

Daniel and Toro s (1995) nd ha he c clical elemen in ind s rial prod c ion (IP) is predic i e for common s ock re rns. We adop heir se of IP as one ins r men for he mon hl models. For q ar erl models, e se he c clical componen of real GNP. Beca se he c clical componen s are no obser able, e deri e bo h series b sing he Hodrick–Presco (1997) l er applied rec rsi el . We elabora e on he cons r c ion of o r da a in he ne sec ion.

Le a and L d igson (2001a) pro ide an al erna i e o hese o p -based meas res of he b siness c cle. Le a and L d igson (2001a) demons ra e ha he c clical elemen in he log cons mp ion–aggrega e eal h ra io (CAY) is s rongl predic i e for e cess s ock re rns. This arg men is consis en i h he CCAPM. Le a and L d igson (2001b) es he CCAPM and he CAPM sing CAY as a condi ioning ariable. In heir cross-sec ional es , condi ioning i h CAY s bs an iall impro es he performance of he models. We also incl de CAY as a condi ioning ariable for he q ar erl models.

Lo ghran (1997) and Daniel and Ti man (1997) arg e ha he book- o-marke (B/M) e ec in s ock re rns is largel dri en b a Jan ar e ec , ha is, he B/M e ec is no presen a o her imes of he ear. The basic asse s e se are he Fama and French 25 por folios hich are cons r c ed precisel o incorpora e he B/M and si e e ec s. We se a Jan ar d mm ariable (JAN) o allo prices of risks o di er be een Jan ar and o her mon hs of he ear.

Ano her impor an iss e is he s abili of he model's parame ers. Condi ional models are a rac i e beca se ncondi ional models ma no adeq a el cap re ime- ar ing risk premi ms. B , his approach is no cos less. If he condi ional ersion is correc l speci ed and cap res he d namics in risk premi ms, i ill o perform he ncondi ional model. Ho e er, if he model's implied ime- ar ing risk premi ms are inheren l misspeci ed beca se e choose he rong condi ioning ariable, his false model ma s ill appear o ork ell in small samples since i ses addi ional degrees of freedom. Gh sels (1998) nds ha condi ional models are fragile and ma ha e bigger pricing errors han ncondi ional models.

If he model is correc l speci ed, parame er s abili is no a problem. We se he s pLM es of Andre s (1993) o see he her here are s r c ral shif s in he parame ers. The n ll h po hesis is ha here are no s r c ral shif s. Andre s arg es ha he s pLM es is po erf l agains he al erna i e of a single s r c ral break a an nkno n ime. He also arg es ha e en if his is no he mos in eres ing al erna i e h po hesis, i pro ides a reasonable es of parame er s abili . The LM s a is ics are e al a ed a 5% incremen s be een 20% and 80% of he sample, and he larges is he s pLM s a is ic. The dis rib ion for he s pLM s a is ic is presen ed in Andre s's Table 1.

To keep he es ima ion rac able, e se he 26 por folios as he basic asse s o be priced. We also in es iga e he her he model is rob s o a di eren se of asse s b adop ing Cochrane's approach of scaling re rns. Cochrane (1996) no es ha condi ioning informa ion can be sed o scale re rns as implied b Eq. (1). These scaled re rns can be in erpre ed as he re rns o managed por folios. The por folio manager changes he eigh of each por folio according o he signal he obser es from he condi ioning ariable. To ill s ra e, e m l ipl bo h sides of Eq. (1) b an ariable $\in \Phi$ o ge

$$\mathrm{E}\,(\quad_{+1}\quad_{,+1}) = \quad, \quad \forall\ , \ > 0, \ \forall \quad \in \Phi\ . \qquad (29)$$

B he la of i era ed e pec a ions, e ha e

$$\mathrm{E}(\quad_{+1}\quad_{,+1}\quad) = \mathrm{E}(\quad), \quad \forall\ , \ > 0, \ \forall \quad \in \Phi\ . \qquad (30)$$

Eq. (30) pro ides he or hogonali condi ions for scaled re rns. If he model is rob s o changes in he nderl ing asse s, i sho ld price he ne asse s correc l . Tha is, if he model can price nonscaled re rns , nder he n ll h po hesis ha he parame ers are no asse -sensi i e, he model sho ld price scaled re rns as ell. The es s a is ic is described in Appendi B.

3.

Unless o her ise indica ed, all da a are from he Cen er for Research in Sec ri Prices (CRSP). For he mon hl models, he sample period in 1952 01 o 1997 12, for 552 o al obser a ions. For he q ar erl models, he sample is from 1953 01 o 1997 04, for 180 o al obser a ions. We begin in 1953 01 beca se CAY is onl a ailable af er 1953 01.

3.1.

O r basic eq i asse s are he 25 e cess re rns on he por folios sor ed b si e and book- o-marke ra io ha are calc la ed as in Fama and French (1993). E cess re rns are cons r c ed b s b rac ing he T-bill ra e, and o r en -si h asse is he gross re rn on he T-bill. The pre io s li era re nds ha he 25 B/M and si e por folios are er hard o price correc l beca se he incorpora e bo h si e premi ms and al e premi ms. We req ire he models o price hese e cess eq i re rns and he risk-free ra e, as ell.

Por folios are n mbered 11–55, here he rs n mber refers o he si e q in ile and he second n mber refers o he B/M q in ile. For e ample, 11 is he por folio of he smalles rms i h he lo es B/M, hile 55 is he por folio i h he larges rms and highes B/M. Table 1 pro ides s mmar s a is ics for he 25 por folios for he sample period 1952 01 o 1997 12. I is similar o Table 2 of Fama and French (1993), hich in ol es a shor er sample period from 1963 01 o 1991 12. For o r longer sample, mos a erage re rns are larger, e cep for he lo B/M rms. Since he s andard errors are smaller, he -s a is ics are larger e cep for he lo B/M rms. Table 1 indica es ha here is considerable di erence in he a erage re rns across he 25 por folios. The a erage ann ali ed re rns range from 4.3% for he smalles rms i h lo es B/M ra io o 13.6% for he smalles rms i h highes B/M ra io. Wi hin a si e q in ile, here is a nearl mono onic increase in a erage re rns as B/M increases. Wi hin he B/M q in iles, he a erage re rns o he smalles rms are larger han he a erage re rns o he larges rms, e cep for he lo es

Table 1

S mmar s a is ics for Fama-French 25 por folios

The da a are mon hl re rns on he Fama-French 25 por folios from 1952 01 o 1997 12 in e cess of he one-mon h T-bill ra e. Por folios are n mbered i h inde ing si e increasing from one o e and inde ing book- o-marke ra io increasing from one o e.

Por folios	BM1	BM2	BM3	BM4	BM5
A Me					
SIZE1	0.36	0.77	0.83	1.03	1.13
SIZE2	0.49	0.78	0.96	1.00	1.15
SIZE3	0.59	0.76	0.80	0.97	1.04
SIZE4	0.60	0.60	0.82	0.87	1.02
SIZE5	0.57	0.63	0.68	0.67	0.85
B o					
SIZE1	7.17	6.25	5.56	5.26	5.53
SIZE2	6.49	5.62	5.11	4.85	5.39
SIZE3	5.94	5.04	4.66	4.50	5.14
SIZE4	5.32	4.80	4.61	4.52	5.22
SIZE5	4.54	4.39	4.09	4.24	4.91
C -					
SIZE1	1.18	2.91	3.52	4.58	4.82
SIZE2	1.76	3.25	4.41	4.85	5.03
SIZE3	2.33	3.55	4.05	5.04	4.76
SIZE4	2.64	2.93	4.17	4.50	4.60
SIZE5	2.97	3.36	3.89	3.74	4.07

B/M q in ile, b here is no mono onici in a erage re rns across si e q in iles.

As demons ra ed in Sec ion 2, he eigh ing ma ri for he calc la ion of HJ-dis ance depends onl on he asse s and is he same for di eren models. The eigh ing ma ri is no he same hen e se condi ioning informa ion o scale re rns. Hence, e ha e fo r eigh ing ma rices mon hl and q ar erl nonscaled re rns, and mon hl and q ar erl scaled re rns. Beca se o r main res l s are deri ed from mon hl and q ar erl nonscaled re rns, e foc s primaril on hese o cases. Eq. (18) demons ra es ha he eigh ing ma ri is he es ima e of he in erse of he second momen ma ri of re rns, hich m s be nonsing lar. The condi ion n mbers of he o ma rices of sample second momen s are 13, 548 and 7, 851 for mon hl and q ar erl re rns, respec i el . For mon hl scaled re rns, he condi ion n mber is 10, 264; for q ar erl scaled re rns, he condi ion n mber is 5, 238. This indica es ha in ersion of he ma rices sho ld be ell beha ed.

Cochrane (1996) no es ha one can ransform he eigh ing ma ri sing eigen al e decomposi ion s ch ha $= \Gamma \quad \Gamma'$ here Γ is an or honormal ma ri i h he eigen ec ors of on i s col mns, and is a diagonal ma ri of eigen al es. Then, he HJ-dis ance problem in Eq. (12) can be re ri en as

$$\delta = [\mathrm{E}(\quad - \quad)'\Gamma \quad \Gamma'\mathrm{E}(\quad - \quad)]^{1/2}. \tag{31}$$

The elemen s of he h col mn in Γ can be in erpre ed as eigh s ha are assigned o he basic asse s o form a por folio associa ed i h he h eigen al e in . If here are a fe large eigen al es of i h eigen ec ors ha place large eigh s on onl a fe por folios, he GMM problem ma be choosing parame ers ha are associa ed onl i h a fe por folios. Beca se does no change across models, i is fair o ask he compe ing models o price he same por folios. B , e do an he s r c re of he eigh ing ma ri o be reasonable.

Fig. 1 presen s he por folio eigh s associa ed i h he o larges eigen al es of he mon hl and q ar erl eigh ing ma rices. The eigh s are s andardi ed o s m o one. For mon hl re rns, Fig. 1 demons ra es ha no par ic lar por folio recei es more han ice he eigh of he ne smalles . Fo r por folios, 14, 15, 41, and 42, recei e s bs an ial eigh s, b se eral o her por folios also recei e non ri ial eigh s. Gi en ha here are o her eigen al es ha are also q an i a i el impor an , e concl de ha he eigh ing ma rices for he HJ-dis ance pro ide a fair challenge o he asse pricing models.

3.2. Co o g v

We se e ariables o cap re mo emen s in he prices of risks o er he b siness c cle. For he mon hl models, he c clical par of he na ral

Panel A:

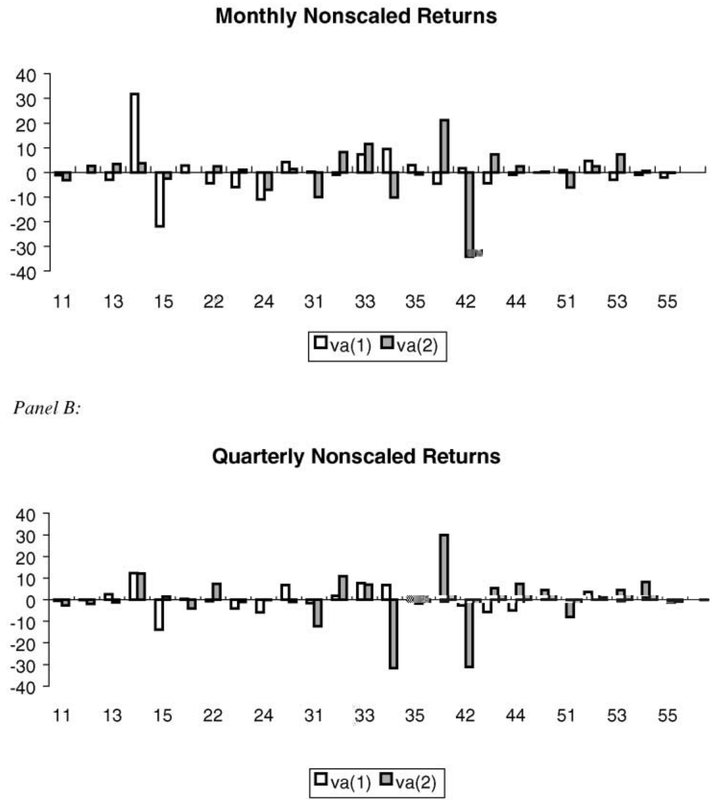


Fig. 1. S andardi ed eigen ec ors of o larges eigen al es of he eigh ing ma ri = $[(1/\ )\sum\ \ ']^{-1}$. The da a are mon hl and q ar erl e cess re rns of he Fama-French 25 por folios and he re rn on he T-bill. Mon hl da a are from 1952 01 o 1997 12. Q ar erl da a are from 1953 01 o 1997 04. The por folio n mbers on he -a is are n mbered i h inde ing si e increasing from one o e and inde ing book- o-marke ra io increasing from one o e. The ec or a(1) and a(2) are he eigen ec ors corresponding o he o larges eigen al es of .

logari hm of he ind s rial prod c ion inde is one condi ioning ariable. The ind s rial prod c ion inde is from he Ci ibase mon hl da a se . The series is a ailable from Jan ar 1947 o April 1999. We se he Hodrick–Presco (1997) l er on he rs e ears o ini iali e he c clical series. The smoo hing parame er is se o be 6,400. Conseq en l , he rs elemen of o r c cle is 1951 12. We hen se he proced re rec rsi el on all a ailable da a o nd he s bseq en elemen s for he c clical series. This me hod g aran ees ha each elemen is in he ime informa ion se . Panel A of Fig. 2 displa s he c clical elemen of log ind s rial prod c ion inde , IP.

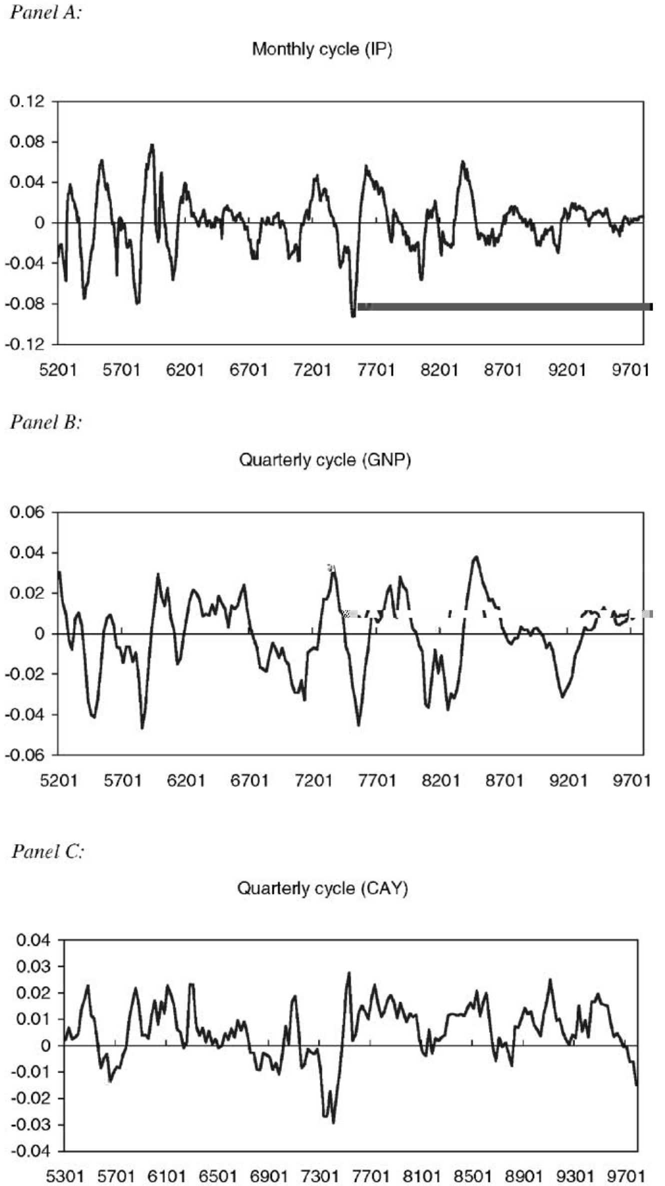


Fig. 2. Time series of hree condi ioning ariables. C cle (IP) is he c clical elemen in mon hl Hodrick–Presco (1997) l ered ind s rial prod c ion. C cle (GNP) is he c clical elemen in q ar erl Hodrick–Presco (1997) l ered GNP. C cle (CAY) is he aggrega e cons mp ion- eal h ra io, deri ed in Le a and L d igson (2001a). Mon hl da a for IP are from 1952 01 o 1997 12. Q ar erl da a for GNP are from 1952 01 o 1997 04, and q ar erl da a for CAY are from 1953 01 o 1997 04.

As men ioned abo e, in mon hl models e also scale he fac ors i h a Jan ar d mm , JAN, ha akes he al e one for each Jan ar and is ero o her ise.-For q ar erl models, JAN akes he al e one for he rs q ar er and is ero o her ise.-

For he q ar erl models, e also scale he fac ors i h he c clical componen of real GNP.-The da a are also from he Ci ibase q ar erl da a se can in erpre he HJ-dis ance as he s andard de ia ion for he leas ola ile elemen in M. In he condi ional case, he N ll model has o fac ors, he cons an and he condi ional . The condi ional N ll model de ermines he her he mo emen in he c cle is an impor an pricing fac or.

The second model is he CAPM. The model SDF has o fac ors, a cons an , and he e cess re rn on he marke por folio. We se he re rn on he al e- eigh ed CRSP inde in e cess of he one mon h risk free re rn, $_{vw}$, as a pro for he e cess re rn on he marke . For he q ar erl model, e compo nd he mon hl marke re rns o prod ce q ar erl re rns, and e s b rac he re rn on he hree-mon h in eres ra e. In he condi ional model of he SDF, here are fo r fac ors he cons an , he , $_{vw}$ and $_{vw} \cdot$.

The hird model is a lineari ed CCAPM. The original CCAPM is nonlinear and req ires a par ic lar form for he ili f nc ion. Ra her han de elop nonlinear models of marginal ili , e simple se cons mp ion gro h, Δ , as he fac or. We se he gro h ra e in real nond rables cons mp ion from CP j9M9 fromZ?Im5Rjqar

The fo r h model is he condi ional CAPM de eloped b Jaganna han and Wang (1996) (hereaf er he JW model).- The JW model is deri ed from he ass mp ion ha he CAPM holds as a condi ional model and ha he re rn on he marke is predic able i h he defa l premi m, PREM, hich is he di erence be een he ield on and corpora e bonds from he Board of Go ernors of he Federal Reser e.- The JW model's ncondi ional form in ol es o be as.- One is he original marke be a.- The o her be a incorpora es aria ion in he marke be a, hich Jaganna han and Wang call be a-premi m sensi i i .-Be a-premi m sensi i i is cap red b aria ion in he defa l premi m.- PREM meas res he ins abili of he marke be a o er he b siness c cle.-Jaganna han and Wang also arg e ha he al e- eigh ed inde is an inadeq a e pro for he marke re rn.-The incl de labor income gro h, LBR, as an addi ional fac or re ec ing a re rn o h man capi al.- Jaganna han and Wang meas reincom-T5qP M' gr-aZP M -ch a, The si h model is a lineari ed ersion of Cochrane's (1996) prod c ion based asse pricing model (described in he ables as COCH). Cochrane arg es ha re rns sho ld be ell priced b he in es men re rn, hich is a complica ed f nc ion of he in es men -capi al ra io and se eral parame ers. B , Cochrane nds ha he in es men gro h ra e performs eq all ell, and e adop he in es men gro h ra e model ins ead of he in es men re rn model. The fac ors are he gro h ra e on real nonresiden ial in es men , GNR, and he gro h ra e on real residen ial in es men , GR. Bo h original series are from Ci ibase. The model has hree fac ors in he ncondi ional model, a cons an , GNR, and GR. The condi ional Cochrane model has si fac ors. The da a are from Ci ibase. Since e onl ha e q ar erl da a for real in es men , e do no comp e a mon hl model in his case.

The abo e si models are all based on e plici economic heories. We also consider o empirical asse pricing models. The are called empirical beca se heir ke pricing fac ors are deri ed from he da a. The se en h model is he Fama-French (1993) hree-fac or model (hereaf er he FF3 model). The rs fac or is he e cess re rn on he marke por folio, VW, as calc la ed abo e. To mimic he risk fac ors in re rns rela ed o si e and B/M ra io, Fama and French (1993) rs sor all s ocks in o o si e por folios, and *g*, he also sor all s ocks in o hree B/M por folios, *g* , , and *o* . Fac or SMB (small min s big) is cons r c ed as he di erence in re rns on and *g*, h s i cap res risk rela ed o si e. Fac or HML (high min s lo) is cons r c ed as he di erence in re rns on *g* and *o* , h s i cap res risk rela ed o he B/M ra io. The ncondi ional model of he SDF has fo r fac ors a cons an , VW, SMB, and HML. We cons r c q ar erl fac ors b compo nding he mon hl fac ors. There are eigh fac ors in he condi ional model.

The eigh h model is he Fama-French (1993) e-fac or model in hich he add a erms r c re fac or and a defa l -premi m fac or o heir hree-fac or model (hereaf er he FF5 model). The erm s r c re fac or, TERM, is he di erence be een he ield on a hir - ear bond and he ield on he one-mon h bill. Defa l risk is he di erence be een he ields on and corpora e bonds (PREM as in JW). We cons r c q ar erl da a b compo nding he mon hl VW, SMB and HML, and e se he hird obser a ion of each q ar er for TERM and PREM. The condi ional model has el e fac ors.

4.

4.1. B o g o

The basic model diagnos ics are presen ed in he se en panels of Table 3. The es ima es of HJ-dis ance are labeled HJ-dis (δ). The - al es of he es $\delta = 0$,

Table 3
S mmar of models sing nonscaled re rns (26 asse s)

The da a are re rns on he Fama-French 25 por folios in e cess of he T-bill ra e and he re rn on he T-bill.- Mon hl da a are from 1952 01 o 1997 12; q ar erl da a are from 1953 01 o 1997 04. C cle (IP) is he c clical elemen in he ind s rial prod c ion inde ; c cle (GNP) is he c clical elemen in real GNP; CAY is from Le a and L d igson (2001a).- JAN is a d mm ariable i h al e one for Jan ar (mon hl models) or rs q ar er (q ar erl models) and ero o her ise.- HJ-dis (δ) is Hansen-Jaganna han dis ance.- - al e for he es $\delta = 0$ calc la ed nder he n ll $\delta = 0$ is $(\delta = 0)$. Ma .-Error is he ma im m ann al pricing error for a por folio i h ann al s andard error of 20% nder he ass mp ion $E(\ ) = E(\ )$. The s andard error for HJ-dis ance nder he al erna i e h po hesis $\delta \neq 0$ is se(δ). The - al e of he op imal GMM es is (J). The - al e of he Wald es ha all condi ional elemen s of * are ero is -Wald(*). The al e of he s pLM s a is ics is s pLM.-An as erisk indica es he model fails he s pLM es a he 5% signi cance le el.-N mber of parame ers is No.-of para.-

MODEL	NULL	CAPM	CCAPM	JW	CAMP	FF3	FF5
A Mo o o o							
HJ-dis (δ)	0.420	0.390	0.429	0.386	0.296	0.323	0.316
$(\delta = 0)$	0.000	0.000	0.000	0.000	0.347	0.000	0.001
Ma .-Error	8.4%	7.8%	8.6%	7.8%	5.9%	6.5%	6.4%
se(δ)	0.051	0.050	0.063	0.052	0.065	0.052	0.055
(J)	0.000	0.000	0.000	0.000	0.194	0.001	0.005
s pLM	216.500*	3.548	4.234	38.290*	193.976*	9.971	58.889*
No.-of para	1	2	2	4	6	4	6
B Mo o o (I)							
HJ-dis (δ)	0.410	0.352	0.389	0.314	0.256	0.302	0.273
$(\delta = 0)$	0.000	0.026	0.041	0.057	0.580	0.010	0.143
Ma .-Error	8.2%	7.1%	7.8%	6.3%	5.1%	6.1%	5.5%
se(δ)	0.054	0.064	0.084	0.050	0.079	0.062	0.062
(J)	0.000	0.269	0.002	0.062	0.534	0.027	0.218
-Wald(*)	0.006	0.003	0.021	0.016	0.486	0.329	0.398
s pLM	10.028	15.963*	9.831	28.254*	73.909*	16.646	40.204*
No.-of para	2	4	4	8	12	8	12
C Mo o o JAN							
HJ-dis (δ)	0.396	0.366	0.367	0.274	0.284	0.287	0.268
$(\delta = 0)$	0.000	0.000	0.057	0.650	0.126	0.101	0.335
Ma .-Error	8.0%	7.3%	7.4%	5.5%	5.7%	5.8%	5.4%
se(δ)	0.060	0.067	0.089	0.086	0.064	0.049	0.067
(J)	0.000	0.000	0.022	0.809	0.065	0.025	0.098
-Wald(*)	0.000	0.165	0.026	0.018	0.962	0.238	0.594
s pLM	5.692	6.244	10.345	52.663*	180.979*	13.470	39.225*
No.-of para	2	4	4	8	12	8	12

MODEL	NULL	CAPM	CCAPM	JW	CAMP	COCH	FF3	FF5
D o o								
HJ-dis (δ)	0.649	0.621	0.619	0.578	0.550	0.626	0.537	0.516
$(\delta = 0)$	0.000	0.000	0.001	0.037	0.016	0.000	0.001	0.018
Ma .-Error	13.2%	12.6%	12.6%	11.8%	11.2%	12.7%	10.9%	10.5%
se(δ)	0.103	0.097	0.108	0.125	0.107	0.113	0.116	0.105

Table 3 (o ɇ)

MODEL	NULL	CAPM	CCAPM	JW	CAMP	COCH	FF3	FF5
(J)	0.001	0.001	0.005	0.083	0.050	0.000	0.010	0.125
s pLM	55.023*	3.671	10.071	31.078*	55.957*	10.026	8.746	52.170*
No. of para	1	2	2	4	6	3	4	6
ɇ E ɇ o ɇ ɇ o ɇ (L GN)								
HJ-dis (δ)	0.642	0.600	0.613	0.543	0.504	0.559	0.452	0.429
(δ = 0)	0.000	0.001	0.000	0.088	0.147	0.108	0.488	0.362
Ma . Error	13.1%	12.2%	12.5%	11.1%	10.3%	11.4%	9.2%	8.7%
se(δ)	0.099	0.082	0.106	0.111	0.104	0.129	0.108	0.099
(J)	0.000	0.011	0.001	0.056	0.101	0.086	0.423	0.254
-Wald(*)	0.219	0.051	0.799	0.013	0.575	0.008	0.111	0.242
s pLM	10.837	11.076	11.578	37.006*	44.640*	9.848	11.285	34.071*
No. of para	2	4	4	8	12	6	8	12
ɇ F ɇ o ɇ ɇ o CA								
HJ-dis (δ)	0.634	0.613	0.608	0.544	0.515	0.623	0.528	0.498
(δ = 0)	0.000	0.000	0.000	0.269	0.099	0.000	0.001	0.011
Ma . Error	12.9%	12.5%	12.4%	11.1%	10.5%	12.7%	10.8%	10.1%
se(δ)	0.099	0.110	0.105	0.154	0.125	0.114	0.105	0.090
(J)	0.001	0.000	0.001	0.428	0.097	0.001	0.003	0.032
-Wald(*)	0.012	0.542	0.253	0.404	0.834	0.609	0.931	0.930
s pLM	14.028*	14.310	7.170	39.171*	40.373*	16.757	20.149	30.937*
No. of para	2	4	4	8	12	6	8	12
ɇ G ɇ o ɇ ɇ o JAN								
HJ-dis (δ)	0.590	0.564	0.582	0.391	0.379	0.510	0.509	0.394
(δ = 0)	0.001	0.001	0.000	0.997	0.975	0.429	0.005	0.870
Ma . Error	12.0%	11.5%	11.9%	8.0%	7.7%	10.4%	10.4%	8.0%
se(δ)	0.135	0.127	0.131	0.239	0.195	0.133	0.129	0.149
(J)	0.011	0.003	0.010	0.997	0.984	0.600	0.004	0.910
-Wald(*)	0.000	0.000	0.006	0.206	0.435	0.001	0.676	0.500
s pLM	8.586	9.181	9.133	32.223*	28.311	11.794	20.144	52.123*
No. of para	2	4	4	8	12	6	8	12

as calc la ed in Appendi A nder he n ll h po hesis ha he r e dis ance is ero, are labeled p(δ = 0). The ma im m ann ali ed e pec ed re rn error from a por folio of he basic asse s based on Eq. (16) is labeled Ma . Error. The ma im m pricing error is he prod c of he HJ-dis ance and he a erage risk-free ra e imes an ass med s andard de ia ion of 20%. The s andard errors for he es ima es of HJ-dis ance are labeled se(δ) and are calc la ed nder he al erna i e h po hesis ha he r e dis ance is no eq al o ero as in Eq. (45) of Hansen and Jaganna han (1997). These s andard errors allo an assessmen of he precision i h hich δ is es ima ed, and he can h s be sed o infer an appro ima e s andard error for he pricing errors in ro hree b m l ipl ing b he a erage risk free re rn and he ass med s andard de ia ion of 20%.

The - al es of he J-s a is ics from op imal GMM es ima es of he models are labeled (J). The - al es of he Wald es s ha he parame ers of he scaled fac ors are all ero are labeled -Wald(*). The al es of he s pLM es s are labeled s pLM, and an as erisk indica es ha he es s a is ic e ceeds he 0.05 cri ical al e aken from Table 1 of Andre s (1993). The n mber of es ima ed parame ers is labeled No. of para.

In ni e samples, in erpre a ion of he HJ-dis ance es ima es and heir associa ed ma im m pricing errors is hampered b he fac ha ero is on he bo ndar of he parame er space. E en if he n ll h po hesis is r e, in ni e samples he es ima ed HJ-dis ance ill be posi i e. Of co rse, if he - al es of he es s a is ics are ell beha ed, false rejec ions of he n ll h po hesis onl occ r he correc percen age of he ime.

The Mon e Carlo e perimen s cond c ed b Ahn and Gadaro ski (1999) indica e ha he e pec ed al e of he HJ-dis ance calc la ed nder he n ll h po hesis ha a hree-fac or model is r e can be q i e large and depends on he n mber of asse s and he n mber of ime periods. From Table 1 of Ahn and Gadaro ski (1999) i h 25 re rns, e nd a erage HJ-dis ances of 0.393 for 160 obser a ions, 0.260 for 330 obser a ions, and 0.174 for 700 obser a ions. Hence, b e rapola ing o o r mon hl sample of 552 obser a ions, e sho ld no be s rprised o see an HJ-dis ance eq al o 0.21, e en ho gh a hree-fac or model is r e. This corresponds o an ann ali ed ma im m pricing error of 4.2%. Similarl , for a q ar erl sample of 180 obser a ions, e sho ld no be s rprised o see an HJ-dis ance eq al o 0.38 i h a ma im m pricing error of 7.7%, e en ho gh he model is r e.

Ahn and Gadaro ski (1999) also in es iga e he empirical si e of he es ha HJ-dis ance eq als ero. For 25 asse s he nd ha 5.5% of heir e perimen s e ceed he 1% cri ical al e i h 160 obser a ions, 2.5% are grea er i h 330 obser a ions, and 1.5% are grea er i h 700 obser a ions. Th s, for o r sample si es, he mon hl model appears o be close o ha l he correc si e of he es if a hree-fac or model is r e, hile he rejec ion ra es for he q ar erl model appear o be oo high.

Panels A–C of Table 3 s mmari e he res l s for he mon hl models. The rs ro of Panel A in Table 3 indica es ha he N ll model, he CAPM, he CCAPM, he JW model, and he FF3 model all ha e HJ-dis ances ha are larger han or eq al o 0.32. The - al es of he es s ha hese dis ances are ero are all less han 0.0001. The ma im m ann ali ed pricing errors from hese models are be een 6.5% and 8.6%. The s andard errors of he HJ-dis ances in ro fo r are all abo 0.05. Hence, he s andard errors of he ma im m pricing errors are all abo 1%. Generall , e nd li le disagreemen be een he Wald es s based on HJ-dis ance or on op imal GMM of he her he pricing errors on he 26 original por folios are join l ero. Conseq en l , e onl repor he J- es s from op imal GMM, and in Panel A of Table 3 e nd e o of he se en models are rejec ed a he 0.001

marginal le el of signi cance or smaller. Campbell's model achie es he smalles HJ-dis ance, and he - al e of he es $\delta = 0$ indica es e canno rejec correc pricing. Th s, he model cap res he si e and B/M e ec s and also prices he risk-free ra e. I is no able ha he same model also passes he J-es . Unfor na el , Campbell's model does no ha e s able parame ers as i fails he s pLM es se erel .

The HJ-dis ance of he FF5 model is smaller han ha of he FF3 model, b i is s ill aro nd 0.30. If e s b rac he small sample bias in he s a is ic of 0.21, disc ssed abo e, e can concl de ha he bias-adj s ed HJ-dis ance is aro nd 0.11 and he ma im m ann ali ed pricing error is aro nd 2.2%. As one migh s spec , he chief di erence be een he FF3 model and he FF5 model comes from he fac ha he T-bill ra e is hard for he FF3 model o price beca se i onl incl des eq i pricing fac ors. To e al a e his conjec re, e did a es hich onl sed gross re rns on he 25 si e and B/M por folios. There ere onl small di erences be een he FF3 model and he FF5 model in ha es , and e co ld rejec correc pricing for bo h models a he 5% marginal le el of signi cance.

Panel B of Table 3 repor s he res l s hen he fac ors of he model SDF's are scaled b c cle(IP). We nd he magni des of HJ-dis ances and he corresponding ma im m pricing errors all shrink signi can l b appro i-ma el 10%, e cep for he N ll model. The - al es for he es of HJ-dis ance eq al ero are no be een 1% and 5%. We es he her he condi ioning informa ion is s a is icall signi can i h a Wald es on he join h po hesis ha he parame ers for all scaled fac ors eq al ero. For he CAPM, he CCAPM and he JW model, he - al es are smaller han 0.023, hich means he scaling ariable IP signi can l cap res ime- ar ing beha ior of risks. Using c cle(IP) red ces HJ-dis ance for all models, and Campbell's model achie es he smalles dis ance, al ho gh here is no signi cance o he parame ers associa ed i h scaling. None of he models pass bo h he es of HJ-dis ance eq al ero and he s pLM es . I is no able ha he CAPM i h scaled fac ors marginall passes bo h he es of HJ-dis ance eq al ero and he op imal GMM es . Again, all res l s from minimi ing HJ-dis ance are similar o ha e nd from he op imal GMM approach.

The fac ha scaled fac or models ha e smaller HJ-dis ances han nonscaled fac or models comes from o so rces. Firs , he condi ioning informa ion red ces he pricing errors b allo ing he prices of risks o ar i h he b siness c cle. Second, b do bling he n mber of parame ers, a scaled fac or model ses addi ional degrees of freedom in he minimi a ion problem and is be er able o he da a. This be er ma be sp rio s, ho gh, as small-sample biases ma orsen. The ne sec ion e amines he de ails of indi id al models.

According o Lo ghran (1997), he Jan ar e ec e plains a s bs an ial par of he B/M e ec . When e allo onl for a Jan ar d mm ariable in addi ion o he cons an erm of he SDF's, here are er fe changes compared o he res l s in Panel A of Table 3. These res l s are no repor ed o sa e space. Panel C of Table 3 repor s res l s i h all fac ors scaled b JAN. This e ec i el separa es he Jan ar obser a ions from he non-Jan ar obser a ions b allo ing di eren fac or risk prices in Jan ar . For he N ll model, he Wald s a is ic for he es ha he JAN parame er eq als ero is 0.0001, hich demons ra es he impor ance of a Jan ar e ec . Allo ing for a Jan ar condi ioning ariable impro es he poin es ima es of HJ-dis ance for all he models. Ne er heless, - al es of he *J* s a is ics indica ed ha he CAPM, he CCAPM, and he FF3 models are s ill rejec ed a he 0.05 le el of signi cance. The mos drama ic impro emen is in he JW model hich no passes all of he es s e cep he s abili es . The Wald es on he impor ance of he scaled fac ors indica es heir join signi cance. There is a sligh impro emen in he performance of he FF3 model al ho gh he join es of he signi cance of he scaled fac ors has a - al e of 0.15. The FF5 model and Campbell's model alread do reasonabl ell i h nonscaled fac ors. Scaling all he fac ors in hese models i h a Jan ar d mm does no appear o add an impor an fac ors since he - al es of he Wald es s are bo h q i e large.

The pre io s li era re picall repor s ei her mon hl or q ar erl models. Some models, s ch as Cochrane's (1996) model, can onl be applied o q ar erl da a beca se of da a cons rain s. In his sec ion e in es iga e he performance of he models i h q ar erl da a. Se eral iss es arise. Firs , ime aggrega ion ma orsen he be een he fac ors and he models b smoo hing he fac ors.[4] Second, marke imperfec ions ha ca se shor - erm de ia ions from he models ma be lessened beca se he re rns are c m la ed. Third, as no ed abo e, he small-sample performance of an model de eriora es i h a smaller n mber of obser a ions. The rs and hird e ec s s gges he performance of he models i h q ar erl da a de eriora es, hile he second fac or allo s for impro emen .

Panel D pro ides he s mmar res l s for he eigh q ar erl models, he se en pre io sl in es iga ed pl s Cochrane's (1996) model. Al ho gh he poin es ima es of he HJ-dis ances are m ch larger for he q ar erl models han he mon hl models, recall from o r disc ssion of Ahn and Gadaro ski (1999) ha al es like 0.38 are o be e pec ed in hese sample si es e en if a hree-fac or model is r e. Ne er heless, he q ar erl HJ-dis ances generall e ceed he a erage of he Ahn and Gadaro ski g res b more han he mon hl es ima es e ceed he corresponding a erage from he Mon e Carlo e perimen s. For e ample, he mon hl FF3 models has an HJ-dis ance of 0.323 and he Mon e Carlo a erage is appro ima el 0.21 for a di erence of

[4] This logic leads Cochrane (1996) o ime a erage mon hl re rns in cons r c ing q ar erl re rns. While e cons r c he q ar erl re rns from he compo nd mon hl re rns as $_{+1} + {}_{+2} + {}_{+3}$, Cochrane (1996) ses $\frac{1}{3}\ {}_{+1} + \frac{2}{3}\ {}_{+2} + {}_{+3} + \frac{2}{3}\ {}_{+4} + \frac{1}{3}\ {}_{+5}$.

0.113. A he q ar erl sampling in er al e nd a di erence of 0.537 − 0.38 = 0.157. Using his bias-adj s ed al e o calc la e he ma im m pricing error for he FF3 model leads o a al e of 3.2% ra her han he 10.9% repor ed in Panel D.

While he - al es of he es s ha HJ-dis ance eq als ero are all less ha 0.037, recall also ha in his sample si e he as mp o ic - al es probabl nders a e he probabili of a T pe I error as Ahn and Gadaro ski (1999) nd ha 15.7% of heir empirical e perimen s e ceed he 5% as mp o ic cri ical al e in samples of 160 obser a ions. Hence, i seems reasonable o concl de ha he e idence agains he JW model, he FF5 model, and Campbell's model is no par ic larl s rong. Unfor na el hese hree models all fail he parame er s abili es .

In Panel E, e scale all fac ors b he lagged c clical componen of GNP. Incl ding his condi ioning informa ion red ces he magni des of HJ-dis ance and he associa ed ma im m pricing errors b 5–10%. T o models, he FF3 model and Cochrane's, no pass he es of HJ-dis ance eq al ero and he s pLM es , al ho gh Cochrane's model has a considerabl larger δ. Once again he HJ-dis ance es s are consis en i h he res l s from op imal GMM. The es s ha all parame ers for scaled fac ors eq al ero indica e scaling i h GNP does no signi can l impro e he performance of he models. One sho ld keep in mind, ho gh, his is a join es hich ma o ershado he signi cance indi id al parame ers.

An al erna i e q ar erl scaling ariable is he cons mp ion- eal h ra io, CAY, from Le a and L d igson (2001a). The nd ha scaling i h CAY grea l impro es he performance of he CCAPM in pricing he e cess re rns on he 25 Fama-French por folios o er a sample period 1963–1997 hen he re rns are eq all eigh ed. Ho e er, e al a ing he model i h he HJ-dis ance me ric for o r sample of 1953 o 1998 indica es ha scaling i h CAY does no prod ce a no iceable impro emen for he CCAPM. The scaled model fails bo h he es of HJ-dis ance eq al ero and he op imal GMM es . None of he models scaled b CAY passes bo h he es of HJ-dis ance eq al ero and he s pLM es . The Wald es of he impor ance of he scaling parame ers also does no indica e s rong s a is ical signi cance of CAY.

Panel G pro ides res l s hen all he fac ors are scaled b JAN. For he q ar erl models, JAN akes he al e one for he rs q ar er of each ear and he al e ero o her ise. The rs hing o no e is scaling all fac ors i h JAN red ces he magni de of he HJ-dis ance for all models. The JW model, Campbell's model, and he FF5 model all ha e - al es for he es of HJ-dis ance eq al ero abo e 80%. The ann ali ed pricing errors for hese hree models also are no less han or eq al o 8%, hich is in he range of correc pricing gi en he bias disc ssed abo e. S rprisingl , he FF3 model does no pass he HJ-dis ance es and he J es . This is beca se he scaled fac or model is s ill nable o price he small gro h rms. Cochrane's model passes bo h he es of HJ-dis ance eq al ero and he s pLM es . More de ails for his model are pro ided in he sec ion on s ccessf l models.

4.2. Mo e e o o o e o o e

Addi ional informa ion on he performance of he models is a ailable b e amining he model errors and he Lagrange m l ipliers hich are he componen s of δ. To check he her condi ioning informa ion impro es he performance of a model, e rs need o nders and he performance of he original nonscaled fac or model. The a erage model errors from HJ-dis ance es ima es i h a o s andard error band are presen ed in Fig. 3. Since mon hl ncondi ional model errors share er similar pa erns i h he q ar erl model errors, e onl presen mon hl model errors (g) as de ned in Eq. (17). For Cochrane's model, e repor q ar erl model errors.[5]

In Panel A of Fig. 3, he model errors for he N ll model range from −0.01% for he T-bill o 1.15% per mon h for por folio 25. Remember ha he rs n mber of a por folio inde es he si e q in ile i h increasing n mbers indica ing increases in si e and ha he second n mber of a por folio inde es he book- o-marke ra io i h increasing n mbers indica ing increases in B/M. The B/M e ec is er e iden in Fig. 3 as in each si e q in ile, higher B/M por folios ha e larger a erage pricing errors. As e increase across si e q in iles, here is less dispersion in he pricing errors b no par ic larl prono nced decrease in a erage pricing errors. The model nder-es ima es he re rns on all por folios e cep he T-bill ra e.

Panel B of Fig. 3 demons ra es ha he CAPM correc l prices he larges si e por folios, b i ends o nder-es ima e re rns on high B/M por folios and o o er-es ima e re rns on lo B/M por folios. The model errors range from −0.50% per mon h for por folio 11 o 0.45% per mon h for por folio 15.

The CCAPM is presen ed in Panel C of Fig. 3. I has a pa ern er similar o he N ll model, hich is consis en i h he correla ion of 0.93 be een he adj s men , $- \tilde{} = \tilde{\lambda}'$, o he N ll model and he adj s men o he CCAPM o make i a correc SDF. High B/M rms are more se erel nderpriced b he CCAPM han b he CAPM.

The JW model is presen ed in Panel D of Fig. 3. I has a er similar pa ern o he CAPM e cep he o er-es ima ion for lo B/M por folios is sligh l smaller. This is no s rprising in ligh of he correla ion of 0.99 be een he adj s men s o he CAPM and o he JW model.

Panel E of Fig. 3 repor s he pa ern for Campbell's pricing errors. The model considerabl a en a es he B/M e ec . The a erage errors range from

[5] We also e amined model errors from minimi ing he eq al- eigh ed s m of sq ared pricing errors, ha is sing an iden i ma ri as he eig hing ma ri . The pa erns of errors across he ario s models are q i e similar o he errors in Fig. 3 and are conseq en l no repor ed.

−0.28% o 0.30%. Par of he abili of he model o pass he es of HJ-dis ance eq al ero arises from i s increased s andard errors rela i e o he CAPM. Al ho gh δ can be compared across models, he - al es of he es s are no comparable beca se he are based on he eigen al es of A in Appendi A hich depend on he pricing fac ors, he ariance of pricing errors, and he n mber of parame ers.

Panel F presen s he pricing errors in Cochrane's q ar erl model hich shares he same magni de and pa ern as he q ar erl CAPM, hich is no presen ed. There is a dis inc B/M e ec as in he mon hl CAPM. The correla ion be een he adj s men o Cochrane's model o make i a correc pricing model and he adj s men o he q ar erl CAPM is 0.97.

The FF3 model is presen ed in Panel G. The presence of he o fac ors SMB and HML in addi ion o he marke re rn considerabl dampens he B/M e ec presen in Panel B. No here is no par ic lar pa ern for he model errors. The are sca ered aro nd he ero a is. The FF3 model o erpredic s he a erage re rns for bo h he smalles rms and he larges rms, b especiall he small gro h s ocks (smalles rms i h lo B/M ra ios). The FF5 model in Panel H has a similar pa ern o he FF3 model, e cep i red ces he pricing errors sligh l . The correla ion of he adj s men s o he o models is 0.98.

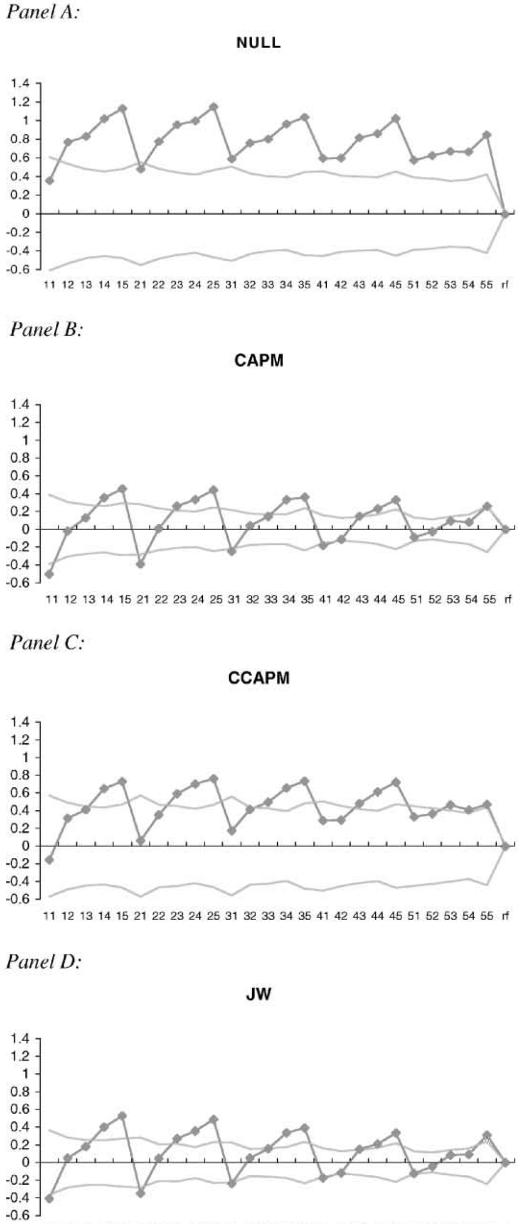
All models share one common charac eris ic, he do no misprice he T-bill ra e. Model errors for he T-bill ra e are al a s aro nd ero.

4.3. I g o

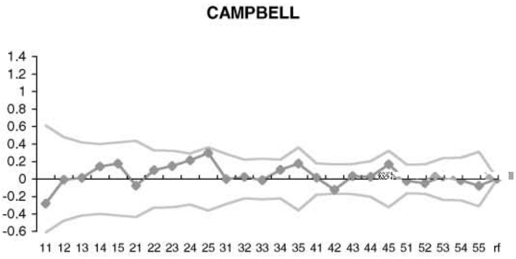
Since e ha e 21 mon hl models and 32 q ar erl models, e canno displa all he parame er es ima es, b e repor res l s for "in eres ing models". We de ne "in eres ing" as a model ha a leas marginall passes he es of HJ-dis ance eq al ero a he 1% marginal le el of signi cance. We also req ire ha he scaling parame ers for an in eres ing scaled fac or model are join l signi can a he 5% le el. Beca se inference abo he alidi of he models based on he es of HJ-dis ance eq al ero is al a s similar o inference based on he J es from op imal GMM, passing he J es is implici l also a cri erion. In o al e ha e 12 models sa isf ing bo h

▶

Fig. 3. Model errors for mon hl models i h nonscaled fac ors. The da a are mon hl and q ar erl e cess re rns of he Fama-French 25 por folios o er he T-bill ra e and he re rn on he T-bill. Mon hl da a are from 1952 01 o 1997 12. Q ar erl da a are from 1953 01 o 1997 04. The por folio n mbers on he -a is are n mbered i h inde ing si e increasing from one o e and inde ing book- o-marke ra io increasing from one o e. The diamonds are he model errors, as de ned in Eq. (17), and he n mbers are in mon hl (q ar erl from Cochranes's model) percen . The o o her lines pro ide a o s andard error band.


Panel A:
NULL
Panel B:
CAPM
Panel C:
CCAPM
Panel D:
JW
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
11 12 13 14 15 21 22 23 24 25 31 32 33 34 35 41 42 43 44 45 51 52 53 54 55 rf

Panel E:



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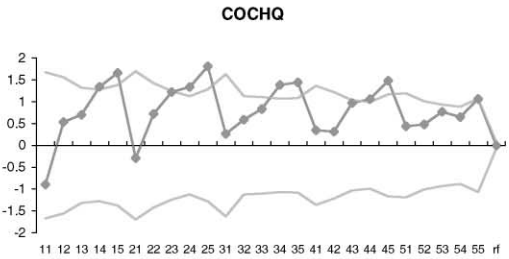

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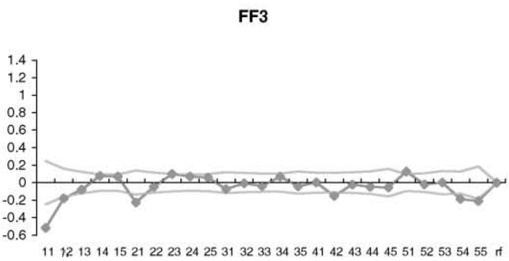


Panel H:

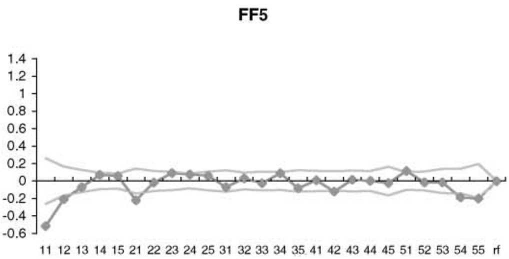


Fig. 3. (*o*)

condi ions. In addi ion e pro ide informa ion on he mon hl FF3 model i h nonscaled fac ors for comparison. This sec ion rs disc sses mon hl models, hen q ar erl models.

Table 4 repor s parame er es ima es from minimi ing he HJ-dis ance meas re for he in eres ing models. Each panel has o par s. The rs par presen s es ima es for as in Eq. (3). If $_1$ for one fac or is signi can l di eren from ero, hen ha fac or is an impor an de erminan of he pricing kernel. The second par of each panel presen s es ima es for he prices of risks, Λ, as in Eq. (4). I pro ides informa ion on he her he fac or risk prices signi can l in ence he e pec ed re rns.

The rs model is he mon hl CAPM i h fac ors scaled b IP. The model marginall passes he es of HJ-dis ance eq al ero i h a - al e of 0.026. Bo h $_{VW}$ and IP are signi can de erminan s of he correc pricing kernel, hile he in erac ion be een he o ariables is no signi can . Th s, he b siness c cle in ence speci ed b IP is an impor an elemen missing from he CAPM. The same o fac ors ha e signi can prices of risks i h posi i e signs. Th s, a posi i e co ariance i h he marke or he s a e of he b siness c cle increases he req ired ra e of re rn. The fac ha IP helps o e plain he B/M and si e e ec s ma arise as in he frame ork of Jaganna han and Wang (1996) beca se IP co ld be a pro for be a-premi m sensi i i . The fac ha $_{VW} \cdot$ IP is no impor an indica es ha allo ing he price of marke risk o change across he b siness c cle is no an impor an de erminan of he cross sec ion of re rns. Panel A of Fig. 4 repor s he model's pricing errors, i h i s nonscaled co n erpar . Mos of he impro emen in pricing from adding IP and $_{VW} \cdot$ IP o he CAPM occ rs for lo B/M por folios, and he bigges impro emen is for he smalles gro h rms. As si e increases, he impro emen becomes smaller. Ho e er, he scaled fac or model does no elimina e ei her he B/M or si e e ec s. The mon hl CAPM i h fac ors scaled b IP also does no pass he s pLM es a he 5% le el indica ing ha he es ima es ma be ns able.

The second mon hl model is he CCAPM i h fac ors scaled b IP. Parame er es ima es are repor ed in Panel B of Table 4. The es of HJ-dis ance eq al ero is passed i h a - al e of 0.041. The parame ers associa ed i h Δ , IP and Δ · IP are all s a is icall signi can elemen s of he pricing kernel. The es ima es for fac or risk prices indica e ha bo h Δ and IP signi can l in ence he e pec ed re rns on he nderl ing 26 por folios i h economicall sensible signs. Re rns ha co ar posi i el i h ei her cons mp ion gro h or he b siness c cle ha e higher req ired ra es of re rn.

The mon hl CCAPM i h fac ors scaled b JAN also sa is es bo h condi ions for being "in eres ing" i h a - al e for he es of HJ-dis ance eq al ero of 0.057. The parame er es ima es are pro ided in Panel C of Table 4. Onl he in erac ion be een Δ and JAN is s a is icall signi can for bo h he pricing kernel and prices of risk. While his res l li erall implies ha he cons mp ion gro h ra e is impor an onl in Jan ar , an al erna i e in erpre a ion is ha he re rn charac eris ics of he nderl ing 26 por folios are mos e iden in Jan ar . The pricing errors for he o scaled fac or ersions of he CCAPM oge her i h he nonscaled fac or benchmark are gi en in Panel B of Fig. 4. When he fac ors are scaled b IP, he impro emen s mos l in ol e a red c ion of he errors for he high B/M por folios b 0.1–0.2% per mon h hich a ens he pricing errors rela i e o he nonscaled CCAPM. When he fac ors are scaled b JAN, bo h he si e e ec and he B/M e ec are smaller and he line connec ing he pricing errors is some ha a er.

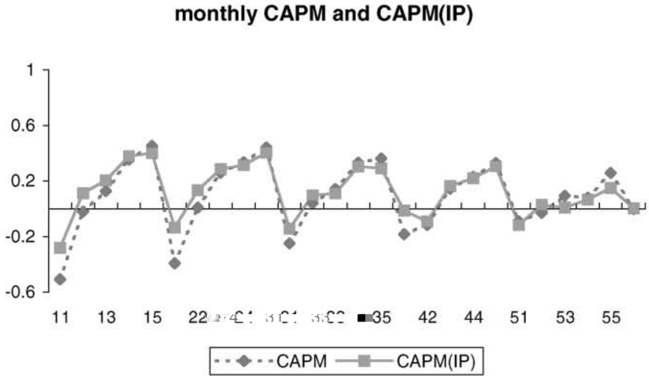
Panel D of Table 4 repor s he parame er es ima es for he mon hl JW model i h fac ors scaled b IP. The - al e for he es of HJ-dis ance eq al ero is 0.057. The signi can de erminan s of he pricing kernel are $_{VW}$ and $_{LBR} \cdot$ IP. The same o fac or risk prices along i h ha of $_{PREM} \cdot$ IP signi can l a ec risk premi ms. Panel E of Table 4 presen s he parame er es ima es for he mon hl JW model i h fac ors scaled b JAN. The - al e of he es of HJ-dis ance eq al ero is 0.650. From he parame er es ima es, bo h $_{LBR}$ and $_{LBR} \cdot$ JAN are signi can de erminan s of he model's pricing kernel. The parame ers indica e ha he fac or risk price of he labor income gro h ra e is di eren in Jan ar $(-0.28 + 0.13 = -0.15)$ han o side of Jan ar (-0.28). The pricing errors of hese o models oge her i h he nonscaled JW benchmark model are presen ed in Panel C of Fig. 4. When he fac ors are scaled b IP, he pricing errors are smaller for bo h small rms and high B/M rms. Th s IP helps dampen bo h he si e e ec and he B/M e ec . When he fac ors are scaled b JAN, he pricing errors are e en smaller, as in he CCAPM abo e. Ho e er, nei her of he models passes he s pLM es .

Campbell's model i h nonscaled fac ors is repor ed in Panel F of Table 4. The model passes he es of HJ-dis ance eq al ero i h a - al e 0.347. Bo h he di idend ield, DIV, and he erm premi m, TRM, are s a is icall signi can de erminan s of he pricing kernel. The second par of Panel F indica es ha hree ariables, $_{VW}$, DIV, and TRM, ha e s a is icall signi can prices of risks. Nei her labor income nor he rela i e bill ra e is impor an in ei her he pricing kernel or he prices of risks. Panel D of Fig. 4

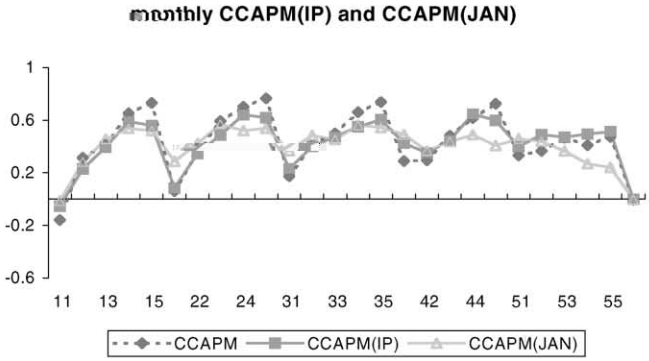
▶

Fig. 4. Pricing errors for in eres ing models. The da a are mon hl and q ar erl e cess re rns of he Fama-French 25 por folios o er he T-bill ra e and he re rn on he T-bill. Mon hl da a are from 1952 01 o 1997 12. Q ar erl da a are from 1953 01 o 1997 04. The por folio n mbers on he -a is are n mbered i h inde ing si e increasing from one o e and inde ing book- o-marke ra io increasing from one o e. Pricing errors are de ned in Eq. (27), and he n mbers are in mon hl (q ar erl) percen .

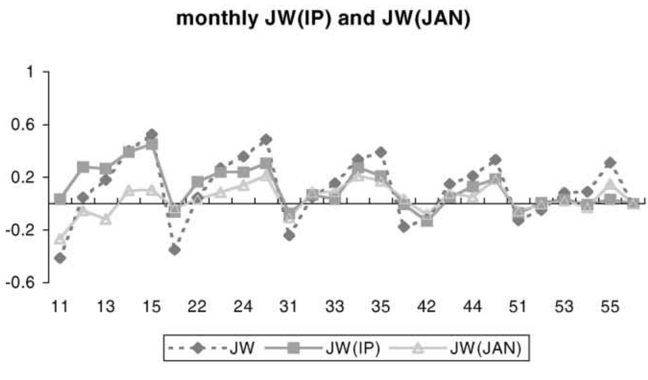
Panel A:



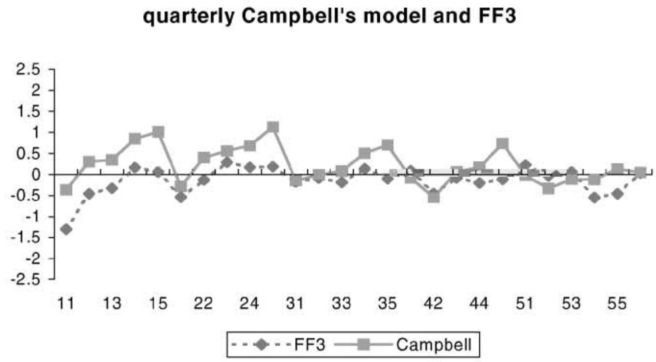
Panel B:



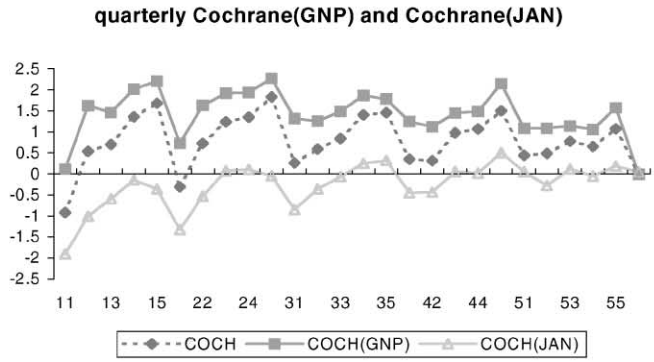
Panel C:



Panel G:



Panel H:



Panel I:

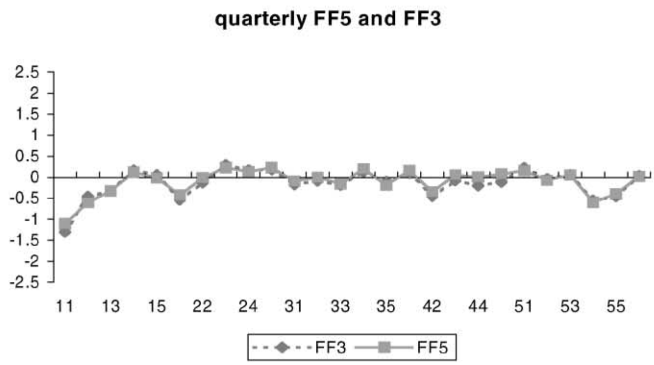


Fig. 4. (o e)

Table 4
Parame ers es ima es of in eres ing models

The da a are re rns on he Fama-French 25 por folios in e cess of he T-bill ra e and he re rn on he T-bill. Mon hl da a are from 1952 01 o 1997 12; q ar erl da a are from 1953 01 o 1997 04. The es ima ed parame ers, ˆ, are he fac or prices de ned in Eq. (3). The es ima ed parame ers, $\hat{\Lambda}$, are he be a risk prices de ned in Eq. (4). The s andard errors for he parame er es ima es are pro ided in he ro s labeled se.

A Mo CA M o I

	Cons an	VW	IP	VW * IP
Parame ers of he pricing kernel				
ˆ	1.03	−0.04	−0.34	0.02
se	0.05	0.02	0.12	0.03
Fac or risk prices				
$\hat{\Lambda}$		0.66	2.16	0.58
se		0.27	0.74	2.75

B Mo CCA M o I

	Cons an	Δ	IP	Δ * IP
Parame ers of he pricing kernel				
ˆ	1.14	−0.75	−0.28	0.22
se.	0.10	0.36	0.11	0.12
Fac or risk prices				
$\hat{\Lambda}$		0.43	1.38	−0.49
se		0.21	0.65	0.55

C Mo CCA M o JAN

	Cons an	Δ	JAN	Δ * JAN
Parame ers of he pricing kernel				
ˆ	1.05	−0.12	0.58	−3.93
se	0.06	0.37	0.90	1.62
Fac or risk prices				
$\hat{\Lambda}$		0.26	0.02	0.20
se		0.22	0.06	0.08

D Mo J ' o o I

	Cons an	VW	PREM	LBR	IP	VW * IP	PREM * IP	LBR * IP
Parame ers of he pricing kernel								
ˆ	1.38	−0.04	−0.66	0.68	0.38	0.00	−0.40	−0.40
se	0.68	0.02	0.64	0.71	0.38	0.03	0.31	0.22
Fac or risk prices								
$\hat{\Lambda}$		0.65	0.05	−0.05	1.01	0.80	1.72	1.09
se		0.28	0.12	0.13	0.98	2.68	1.02	0.41

Table 4 (*o*)

J C ' o o o

	Cons an	vw	LBR	DIV	RTB	TRM
Parame ers of he pricing kernel						
ˆ	0.22	0.00	0.10	0.28	−0.20	−0.56
se	1.00	0.02	0.16	0.27	2.64	0.22
Fac or risk prices						
$\hat{\Lambda}$		1.52	−0.13	−0.28	−0.03	0.85
se		0.79	0.37	0.24	0.02	0.34

K Co ' o o GN

	Cons an	NRINV	RINV	GNP	NRINV∗GNP	RINV∗GNP
Parame ers of he pricing kernel						
ˆ	0.92	−0.01	−0.16	0.12	−0.04	−0.09
se	0.27	0.16	0.07	0.22	0.07	0.04
Fac or risk prices						
$\hat{\Lambda}$		0.33	1.76	0.03	0.86	5.33
se		0.85	1.31	0.58	1.21	3.24

L Co ' o o JAN

	Cons an	NRINV	RINV	JAN	NRINV∗JAN	RINV∗JAN
Parame ers of he pricing kernel						
ˆ	1.41	−0.24	0.09	−1.44	0.90	−0.19
se	0.21	0.17	0.07	0.53	0.37	0.15
Fac or risk prices						
$\hat{\Lambda}$		−0.63	−1.38	0.15	−1.25	−0.03
se		0.75	1.44	0.08	0.59	0.61

M FF5 o o

	Cons an	vw	SMB	HML	TERM	PREM
Parame ers of he pricing kernel						
ˆ	1.23	−0.05	0.00	−0.06	−0.21	1.25
se	0.52	0.02	0.02	0.02	0.11	0.78
Fac or risk prices						
$\hat{\Lambda}$		1.51	0.58	1.12	0.23	−0.06
se		0.79	0.42	0.41	0.51	0.10

repor s he model's pricing errors along i h he errors from he FF3 model as a comparison. No si e e ec is apparen and Campbell's model prices he small gro h rms be er han he FF3 model. While a B/M e ec is presen in he pricing errors of Campbell's model, i s magni de is no large. O erall, he pricing errors for Campbell's model are no bigger han hose of he FF3 model, hile he la er model is cons r c ed o price he si e e ec and B/M e ec . Ho e er, Campbell's model fails he s pLM es . Th s, he parame er es ima es ma be ns able and sho ld be sed ca io sl .

The las mon hl models e repor are FF3 i h nonscaled fac ors and FF3 i h fac ors scaled b JAN. FF3 is repor ed beca se i is so idel sed, and e an o e amine ho i prices he si e and B/M e ec s, hich i is cons r c ed o do. I does no pass he es of HJ-dis ance eq al ero. Parame er es ima es for FF3 are presen ed in Panel G of Table 4. I is some ha s rprising ha onl $_{VW}$ and HML are impor an for he pricing kernel, and he are also signi can l priced risk fac ors. Panel E of Fig. 4 pro ides he pricing errors for FF3. The problem por folios are he lo es B/M i h smalles and second smalles si es, hich are o erpriced b he model. Th s, he fac or SMB canno adeq a el cap re he si e e ec in he por folios, and SMB is no signi can l priced in he ncondi ional ersion hen risk prices are held cons an .

The mon hl FF3 i h fac ors scaled b JAN is repor ed in Panel H of Table 4. I passes he es of HJ-dis ance eq al ero i h a - al e of 0.101. From he parame er es ima es, $_{VW}$, SMB and SMB · JAN are impor an fac ors for he pricing kernel. For he prices of risks, $_{VW}$, HML and SMB · JAN are signi can . This is consis en i h he ie ha he si e e ec is primaril a Jan ar e ec as he prices of risks for $_{VW}$ and HML are essen iall he same across he models i ho and i h scaling b he Jan ar d mm . As men ioned in he pre io s sec ion, if he B/M e ec occ rred mainl in Jan ar , and HML e plained he B/M e ec , HML o ld no be priced o side Jan ar . Th s, he res l s ell s ei her here is s ill a signi can B/M e ec o side of Jan ar or here are some o her risks hich can be priced b HML. We also e amine he pricing errors o see he her scaling b JAN reall impro es on he performance of he FF3 model in an in eres ing a . In he Panel E of Fig. 4, e nd ha scaling he FF3 fac ors i h JAN ac all red ces he pricing errors b 0.2% for he smalles gro h s ocks. Since he FF3 model alread cap res he B/M e ec reasonabl ell, JAN does no impro e his dimension. Bo h models pass he s pLM es .

The rs q ar erl model is he JW model. I marginall passes he es of HJ-dis ance eq al ero i h a - al e 0.037. The parame er es ima es are presen ed in Panel I of Table 4. Onl $_{LBR}$ is s a is icall signi can in he pricing kernel. For he prices of fac or risks, $_{LBR}$ is also signi can i h a posi i e sign. In addi ion, he price of marke risk is marginall signi can , b $_{PREM}$ is no priced in con ras o Jaganna han and Wang (1996). The pricing errors of he JW model are repor ed in Panel F of Fig. 4 oge her i h he q ar erl FF3 model i h nonscaled fac ors as a benchmark. Bo h he si e e ec and he B/M e ec are e iden in he JW pricing errors, hich range from 0.5% o 2% per q ar er. These pricing errors are q i e large compared o hose of he FF3 model. Th s, he q ar erl JW model passes he HJ-dis ance es no beca se i has small pricing errors b beca se i has larger s andard errors. Hence, o r q ar erl ersion of he JW model i h nonscaled fac ors is no an economicall in eres ing model. I also fails he s pLM es indica ing ha he parame er es ima es ma be ns able.

The second q ar erl model is Campbell's model i h nonscaled fac ors. The es of HJ-dis ance eq al ero has a - al e 0.016. Panel J of Table 4 pro ides he parame er es ima es, and as in he mon hl models, he erm premi m is impor an in he pricing kernel. Bo h marke risk and erm premi m risk are priced fac ors for he risk premi ms. The pricing errors are repor ed in Panel G of Fig. 4 oge her i h he benchmark FF3. The pa ern of he errors is er similar o he mon hl errors in Panel D. Campbell's model impro es on he smalles gro h por folio, b i has an e iden B/M e ec . I also fails he s pLM es .

The hird q ar erl model is Cochrane's model i h fac ors scaled b he c clical elemen in lag GNP. The parame er es ima es are gi en in Panel K of Table 4. For he pricing kernel, bo h RINV and RINV · GNP are impor an , and bo h ha e marginall signi can prices of risks. This is consis en i h Cochrane (1996) ho demons ra es he impor ance of residen ial in es men . The HJ-dis ance meas re drops from 0.626 for Cochrane's nonscaled fac or model o 0.559 for i s scaled fac or model. In all of he models disc ssed abo e, he scaled-fac or models perform be er han nonscaled models in he sense of HJ-dis ance, and e con rm ha he scaling fac ors are economicall in eres ing b looking a he pricing errors and parame er es ima es. Ho e er, for Cochrane's model, he impro emen in HJ-dis ance does no ac all come from he impro emen s on pricing errors. This can be seen in Panel H of Fig. 4. The pricing errors of he nonscaled model sho a dis inc pa ern of si e and B/M e ec s. The scaled fac or model shif s mos of he pricing error p ard b 0.5–1%. There is impro emen onl for he rs por folio. The smaller HJ-dis ance for he scaled fac or model arises beca se he addi ional free parame ers make i easier for he scaled-fac or model o sol e he minimi a ion problem i h he par ic lar eigh ing ma ri . This is signi can s a is icall , b i is no in eres ing economicall .

Panel L of Table 4 repor s he q ar erl Cochrane model i h fac ors scaled b JAN. Bo h JAN and NRINV · JAN are impor an for he pricing kernel, and he same o fac ors also ha e signi can prices of risks. B looking a Panel H of Fig. 4, e nd af er con rolling for he Jan ar e ec , he pricing errors are shif ed do n ard b 1–1.5%, hich is a big impro emen for al e rms. The B/M e ec is mi iga ed b s ill presen . Th s e concl de ha he impro emen in HJ-dis ance arises from an impro emen of pricing errors. Bo h Cochrane's scaled fac or models are s able, and he bo h pass he s pLM es .

The q ar erl FF5 model i h nonscaled fac ors is pro ided in Panel M of Table 4. I passes he es of HJ-dis ance eq al ero i h - al e 0.018. From he parame er es ima es, e nd ha VW and HML are de erminan s of he pricing kernel, as in he FF3 model, b he o macro fac ors, TERM and PREM are also signi can de erminan s of he pricing kernel. The o macro fac ors do no ha e signi can prices of risks. The pricing errors from FF5 in Panel I of Fig. 4 are almos he same as hose in FF3. There are onl small impro emen s on he smalles gro h por folios. Unfor na el , he o addi ional macro fac ors bring ins abili in o he model as i fails he s pLM es .

There is one las iss e o no e. All of he models do ell in pricing he gross re rn of he T-bill. This implies ha al ho gh he minimi a ion problem does no p a par ic larl large eigh on he T-bill re rn, i does no ignore i ei her. O hers, s ch as Le a and L d igson (2001b) and Jaganna han and Wang (1996), onl incl de s ock por folios and ha e big es ima es for he ero-be a ra e. We es ima e he ero-be a ra e for each model. For mon hl models, he ra e is aro nd 0.4% per mon h; for q ar erl models, i is aro nd 1.8% per q ar er. We belie e hese es ima es are more reasonable.

4.4.

We no ed abo e ha he sol ion for he HJ-dis ance from he N ll model pro ides he leas ola ile elemen of he se of r e s ochas ic disco n fac ors, M. From Eq. (9) e kno ha $\tilde{} = - \tilde{\lambda}'$, and Eq. (10) pro ides he es ima ed al es of he Lagrange m l ipliers. The s andard errors of he Lagrange m l ipliers are fo nd from Eq. (24). These al es for he N ll model are presen ed in Table 5 for he mon hl and q ar erl da a.

The Lagrange m l ipliers can be in erpre ed as por folio eigh s on he basic asse s. The are he prod c of he HJ-dis ance eigh ing ma ri and he ec or of a erage pricing errors from he model. As bo h he eigh s and he errors di er across asse s and beca se here is correla ion across he elemen s of he m l ipliers, he in erpre a ion of he indi id al signi cance of he m l ipliers is bes done i h ca ion. Ne er heless, for mon hl da a, e nd ha por folios 11,14,42, and 53 as ell as he risk free re rn ha e s a is icall signi can m l ipliers hen he indi id al coe cien s are e al a ed a he 5% cri ical le el. For q ar erl da a, hese same por folios pl s por folios 41 and 54 are also impor an . The impor ance of hese por folios is consis en i h he obser a ion ha i hin each si e q in ile, here is a leas one large spread posi ion in hich one of he Lagrange m l ipliers is a large

Table 5
λ for mon hl and q ar erl n ll models

The da a are re rns on he Fama-French 25 por folios in e cess of he T-bill ra e and he re rn on he T-bill. Mon hl da a are from 1952 01 o 1997 12; q ar erl da a are from 1953 01 o 1997 04. Por folios are n mbered i h inde ing si e increasing from 1 o 5 and inde ing book-o-marke ra io increasing from 1 o 5. The Lagrangian M l ipliers, λ's, are de ned in Eq. (10) and heir s andard errors, se(λ), are de ned in Eq. (26). An as erisk indica es he parame er is signi can a he 5% le el.

	Mon hl		Q ar erl	
Por folio	λ	se(λ)	λ	se(λ)
11	−6.35*	1.73	−5.42*	1.72
12	−3.83	2.41	−3.60	2.32
13	−1.75	3.27	−3.32	3.52
14	8.76*	4.24	10.02*	4.69
15	3.72	3.71	−2.45	4.24
21	−3.66	2.65	−3.93	2.69
22	−0.09	3.15	5.02	3.32
23	6.94	3.73	5.59	3.48
24	3.40	3.75	5.28	3.83
25	2.56	3.27	4.56	3.43
31	−2.75	3.17	−3.53	3.22
32	0.02	3.67	0.17	4.05
33	−3.72	3.87	−7.36	4.36
34	5.85	3.82	4.79	4.35
35	−0.29	2.71	1.92	2.58
41	6.92	3.62	9.90*	4.03
42	−10.59*	3.95	−11.97*	4.18
43	0.09	3.66	0.91	3.98
44	−0.67	3.10	−4.63	3.64
45	0.36	2.33	2.33	2.57
51	1.78	2.43	0.28	2.39
52	−0.25	3.11	1.21	3.15
53	5.65*	2.70	5.48*	2.77
54	−4.22	2.64	−6.21*	2.85
55	−0.30	1.67	−0.16	1.72
$_f$	−0.18*	0.02	−0.42*	0.06

posi i e n mber and ano her one close b is a large nega i e n mber. For he small rms, he por folio posi ions indica e being long high B/M rms and shor lo B/M rms. S mming i hin a si e q in ile re eals ha one o ld be primaril long he second and shor he fo r h si e q in iles. Beca se he spread posi ions are probabl associa ed i h a single so rce of risk, i appears ha here are essen iall fo r so rces of signi can eq i risk in hese 25 por folios.

4.5. Co g o o o o

An al erna i e a o compare models is o incl de he fac ors of se eral models sim l aneo sl in o he model of he pricing kernel and perform an e cl sion es asking he her he second se of fac ors is necessar in he presence of he rs . This sec ion performs a limi ed comparison beca se he large dimensionali of he fac ors and scaled fac or makes s ch a comparison impossible.

In he anal sis abo e, bo h he Campbell model and he Fama-French hree-fac or model are reasonabl s ccessf l. B incl ding he o addi ional FF3 fac ors, SMB and HML, in he pricing kernel of he Campbell model, one can es he her he are signi can addi ional de erminan s of he pricing kernel. The res l s of his anal sis are presen ed in Panel A of Table 6. No ice ha none of he indi id al coe cien s is signi can a he 0.05 le el of signi cance, in s rong con ras o he res l s of he indi id al models. This is an indica ion of m l icollineari . Correla ion across he fac ors also makes he e cl sion es s inconcl si e. The - al e of he Wald es ha he parame ers associa ed i h SMB and HML are ero is 0.135 indica ing ha hese fac ors are nnecessar once he Campbell fac ors are presen , b he comparable es ha he FF3 model does no need he fo r addi ional fac ors of Campbell's model has a - al e of 0.215. Th s, since he fac ors of he respec i e models are signi can hen incl ded indi id all , e can concl de ha he same basic informa ion is cap red in di eren a s b he o models.

To a oid problems i h m l icollineari , Campbell (1996) or hogonali es he fac ors and scales hem o ha e he same ariance as he marke re rn. The rs fac or is he marke re rn, he second is he par of labor income ha is no e plained b he marke re rn, he hird is he par of he di idend ield ha is no e plained b he marke re rn and labor income, and so on. When e place he o Fama-French fac ors af er he e Campbell fac ors, e ask he her he par s of SMB and HML ha canno be e plained b he Campbell fac ors are signi can de erminan s of he pricing kernel. The res l s are presen ed in Panel B of Table 6.

The coe cien s on VW, DIV, TRM, and HML are all more han 1.5 imes heir s andard errors. In par ic lar, e en ho gh HML is placed las in he ordering of ariables, i s - al e remains 0.069. Th s, HML appears o add some independen informa ion o he pricing kernel o er and abo e ha pro ided b he Campbell fac ors.

Panels C and D of Table 6 repor he res l s of a h brid model ha ses hese fo r elemen s i h or hogonali ed fac ors. The h brid model has he smalles HJ-dis ance, 0.285, of an of he es ima ed models, and he es s indica e no e idence agains he model, e cep for he s abili es hich again indica es po en ial problems i h he model.

Table 6
Combining factors of Campbell's model and the Fama-French three-factor model

The data are returns on the Fama-French 25 portfolios in excess of the T-bill rate and the return on the T-bill. Monthly data are from 1952:01 to 1997:12; quarterly data are from 1953:01 to 1997:04. The factors are collected from Campbell's model and FF3. We use Cholesky decomposition to orthogonalize factors in Panels B and C. The parameter estimates, $\hat{b}$, are factor prices defined in Eq. (3). The standard errors for $\hat{b}$ are provided in the row of se. The p-value for the test $\hat{b}=0$ is $p(\hat{b}=0)$. HJ-dist (δ) is Hansen-Jagannathan distance. p-value for the test $\delta=0$ calculated under the null $\delta=0$ is $p(\delta=0)$. Max. Error is the maximum annual pricing error for a portfolio with annual standard error of 20% under the assumption $E(y)=E(\hat{y})$. The standard error for HJ-distance under the alternative hypothesis $\delta\neq0$ is se(δ). The p-value of the optimal GMM test is $p(J)$. The value of supLM statistics is supLM. An asterisk indicates the model fails the supLM test at the 5% significance level. No. of para is the number of parameters.

Factors	Constant	R_{vw}	LBR	DIV	RTB	TRM	SMB	HML
Panel A								
$\hat{b}$	−0.31	−0.02	−0.11	0.43	0.70	−0.38	−0.02	−0.06
se	1.03	0.02	0.33	0.27	3.33	0.26	0.02	0.03
$p(\hat{b}=0)$	0.76	0.35	0.74	0.11	0.83	0.14	0.37	0.07
Panel B								
$\hat{b}$	−0.31	−0.05	−0.03	0.11	0.07	−0.10	−0.01	−0.03
se	1.03	0.01	0.07	0.06	0.07	0.06	0.01	0.02
$p(\hat{b}=0)$	0.76	0.00	0.61	0.09	0.27	0.12	0.46	0.07

Factors	Constant	R_{vw}	DIV	TRM	HML
Panel C					
$\hat{b}$	0.02	−0.05	0.09	−0.14	−0.03
se	0.95	0.01	0.06	0.06	0.02
$p(\hat{b}=0)$	0.99	0.00	0.12	0.02	0.11

HJ-dist (δ)	$p(\delta=0)$	Max. error	se(δ)	$p(J)$	SupLM	No. of para
Panel D						
0.285	0.235	5.7%	0.058	0.144	192.736	5

The intuition of the Campbell model is that any variable that predicts the market return in a multivariate setting is a potential factor that affects the cross-section of asset prices. To determine whether HML arises as a risk factor within this restricted context, we estimated a vector autoregression of the four factors. The results indicate that HML is not an important determinant of the o her hree ariables beca se he smalles - al e associa ed i h he coe cien s on HML in an of he hree eq a ions as 0.336. The HML eq a ion also indica ed ha none of he o her hree ariables is a signi can de erminan of HML, al ho gh here is e idence of o n serial correla ion. Th s, if HML is a risk fac or, i s impor ance m s be e9an o he more general economic s a e ariables of Mer on (1973) ra her han he res ric ions arisn in Campbell's model. Some s pn por for his posi n is pro ided b Lie an Vassalo (2000) and Vassalo (2000) ho arg e ha SMB and HML are risk fac ors ha arise beca se of heir abili o predic f re GDP.

4.6. o

In all of he abo res l s, e ob ain paraman er es ima es an cond c es s sing nonscaled re rnn To e amine he her hese models are rob s , e change he nderlMing asse s from nonscaled re rn o scaled re rn in es iga e he her h models (he rs s age es ima es) can price he scaled re rnn We scale re rnn i h he erm premi m, he di erence in ields be een a 30- ear go ernmen bond and a one- ear go ernmen bond. If a model is able o price he ba asse s (nonscaled re rn he scaled re rn hich he manager in es s di eren amo n dependi erm premi m.

Table 7 pro ides he inforn on hese e perimen s. We se he es ima es ob ainan d from he rs s age b op imal GMM, o calc la e he es of he HJ- dis ance eq al ero fornMj heZP- scaleZPjM-J-dZPs qM9rnMj rnp inMZPaj5Mhe GMMZ

5.

This paper e al a es a of asse pricing e he anomalies nco ered in es ing he CAP common se ofre rn 25 an book- o-marke Fama an French forn,4M4 aZP ' sampl as high as 1.13% per mon h. Wi hin marke por folios ha e higher a erage re

Table 7
Rob s ness es for nonscaled re rns models

The es s are based on re rns on he Fama-French 25 por folios in e cess of he T-bill ra e and he re rn on he T-bill, condi ioned on he erm premi m, he di erence in ields be een a 30- ear go ernmen bond and a one- ear bond. Mon hl da a are from 1952 01 o 1997 12; q ar erl da a are from 1953 01 o 1997 04. The - al es are 1, es of HJ-dis ance = 0 sing parame er es ima es from op imal GMM for corresponding nonscaled re rn models; 2, es of op imal GMM o er-iden i ca ion sing parame er es ima es from op imal GMM for corresponding nonscaled re rn models; 3, es of HJ-dis ance = 0 sing parame er es ima es from minimi ing HJ-dis ance for corresponding nonscaled re rn models.

	NULL	CAPM	CCAPM	JW	CAMP	FF3	FF5
A Mo E M o o							
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0.001	0.003
3	0	0	0	0	0	0	0
B Mo E M o I							
1	0	0	0	0.002	0	0	0
2	0	0.004	0	0.004	0	0.017	0
3	0	0	0	0	0	0	0
C Mo E M o JAN							
1	0	0	0.001	0	0	0	0
2	0	0.001	0.036	0.002	0	0.007	0.086
3	0	0	0.075	0.004	0	0.001	0.001

	NULL	CAPM	CCAPM	JW	CAMP	COCH	FF3	FF5
D E M o o								
1	0	0	0.014	0.002	0	0	0.002	0.003
2	0.003	0.005	0.012	0.006	0	0	0.040	0.049
3	0	0	0.006	0.001	0	0	0.001	0.002

E ,

30.0050.006 0.001 0 00.0010
2 0.001 0.005 0.0060.001

E

35qjj9qj95M’-49 M 0.001ZPq 9Mj 0ZP 9qM4 0ZPq -’M 0.001ZP-9 M1 0.003Z?ImW qR9R?fnP
j 30.0050.0120.001

book- o-marke q in iles, a erage re rns are generall decreasing in si e. The ncondi ional CAPM canno e plain hese re rns.

We consider onl lineari ed ersions of he models, and e e al a e he models i h bo h nonscaled fac ors and scaled fac ors, here he scaling re ec s ei her b siness-c cle mo emen s or a Jan ar d mm . The models are compared sing he me hodolog of Hansen and Jaganna han (1997) ho recogni e ha he es ima ed dis ance be een a model's pricing kernel and he r e pricing kernel also is an es ima e of he ma imal mispricing of a por folio of he asse s. We also e al a e he models sing he op imal GMM es of Hansen (1982). In general, e nd li le disagreemen be een he o es s. Finall , e e al a e he emporal s abili of he parame ers sing he s pLM es of Andre s (1993).

For mon hl models i h nonscaled fac ors, Campbell's (1996) model is he onl model ha passes he es of HJ-dis ance eq als ero, and i s es ima ed HJ-dis ance is also smaller han ha of he Fama-French (1993) hree-fac or model. Onl hree of he e fac ors in he model appear o be impor an he re rn on he marke por folio, he di idend ield, and he erm premi m. The HML fac or of he Fama-French model also has independen informa ion o er and abo e ha pro ided b hese hree fac ors. Unfor na el , he Campbell model fails o pass he s abili es . While he sim la ion s d of Ahn and Gadaro ski (1999) pro ides some s ppor ha he small-sample dis rib ions of he HJ-dis ance es are reliable for o r sample si e, no comparable s d of he small-sample dis rib ions of he s abili es has been cond c ed. Th s, addi ional s d of he Campbell model is desirable. In par ic lar, e e al a e onl he lineari ed ersion of he model.

Scaling he risk fac ors of he models i h he c clical elemen in ind s rial prod c ion as meas red b he Hodrick–Presco (1997) l er impro es he performance of se eral of he models. The CAPM, CCAPM, and Jaganna han and Wang (1996) models all ha e signi can coe cien s on he scaled fac ors. There is also e idence ha pricing in Jan ar is signi can l di eren han pricing o side of Jan ar . For e ample, hen he hree fac ors of he Fama-French (1993) model are en ered i ho scaling, onl he marke re rn and he HML por folio are signi can risk fac ors. When he fac ors are also scaled i h a Jan ar d mm , he marke re rn and he HML por folio re ain heir signi cance and he SMB por folio is signi can in Jan ar . This la er model also passes he s abili es .

Wi h q ar erl da a, none of he models i h nonscaled fac ors passes he es of HJ-dis ance eq al o ero. Ne er heless, he sim la ion res l s of Ahn and Gadaro ski (1999) s gges ha hese res l s sho ld be in erpre ed i h care as he si es of he es s appear o de eriora e in his sample si e. Nei her scaling i h he c clical componen of GNP, as meas red b he Hodrick–Presco (1997) l er, nor scaling i h he cons mp ion- eal h series of Le a and L d igson (2001a) has m ch of an in ence on he res l s.

Addi ionall , none of he models, ei her mon hl or q ar erl appears o be rob s in he follo ing sense.-When e es ima e he parame ers of he models sing he basic re rns and ask he models o price he se of asse s cons r c ed b scaling re rns i h he erm premi m, all of he models fail.-

There are se eral direc ions in hich his s d co ld be e ended.-Firs , e cons r c o r es ima es as if here are no ransac ions cos s or shor -sale cons rain s in asse marke s.-Hanna and Read (1999) nd ha ransac ion

A .

We rs calc la e parame er es ima es from op imal GMM sing he 26 re rns as

$$\hat{} = \arg\min g \; (\; , \;)' \quad {}^{*} g \; (\; , \;). \tag{B.1}$$

Then, nder he n ll ha $\hat{}$ is he r e parame er, he se of scaled re rns sho ld be correc l priced i h $\hat{}$. We calc la e he ne J s a is ics as

$$J = g \; (\quad , \hat{})' v \quad [g \; (\quad , \hat{})]^{-1} g \; (\quad , \hat{}), \tag{B.2}$$

here

$$g \; (\quad , \hat{}) = \frac{1}{} \sum_{=1}^{-1} [(\quad _{+1} \quad)(\hat{}' F_{+1}) - \quad]. \tag{B.3}$$

The J-s a is ic is dis rib ed as a $\chi^2(\;)$ nder he n ll. The degrees of freedom are beca se e ha e or hogonali condi ions, and e do no es ima e an addi ional parame ers. The same arg men applies o HJ-dis ance. Wi h he ne or hogonali condi ions for scaled re rns, e need o calc la e he ne δ and he dis rib ion of δ^2. Since he rs s age es ima es b op imal GMM are no er di eren from hose ob ained from HJ-dis ance es ima ion, e choose o se he es ima es from op imal GMM o calc la e ne HJ-dis ances for he ne scaled asse s.

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