Seeing the Unobservable from the Invisible:
The Role of CO_2 in Measuring Consumption Risk

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Abstract

Although it may be difficult for households to instantaneously adjust their stock of durable goods, they have much more latitude in adjusting the service flow from that stock. In contrast to past studies that assume service flow to be a constant fraction of the stock, we model the utilization of the stock of durable goods to be time-varying. We propose an innovative measure of the unobserved usage of durable goods from carbon dioxide emissions. Emissions provide a convenient aggregation of energy consumption that has become an important complementary input for durable goods consumption in recent decades. We find that the time-varying utilization of durable goods is a valid pricing factor. Our model exhibits a stronger cross-sectional pricing power than the CAPM and several consumption-based capital asset pricing models (CCAPMs), including Yogo's (2006) durable good model. Finally, our model mitigates the joint risk premium and implied risk-free rate puzzle.

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Keywords: Consumption-Based Capital Asset Pricing Model (CCAPM), Durable Goods Service Flow, Carbon Dioxide Emissions

1 Introduction

Households derive utility from consuming durable goods. When utility derived from durable and nondurable consumption is not additively separable, household consumption of durable goods enters the Euler equation and hence affects asset prices. This point has been studied by Yogo (2006). However, in Yogo's study and in other past studies of consumption based capital asset pricing models (CCAPMs) with durable goods, households derive utility from service flow of durable goods that is assumed to be a constant fraction of the stock. Although simple, this assumption is improbable — a household's use of car, gas heater, electric appliance, and many other durable goods, varies from year to year. We develop a CCAPM that allows for time variation in the service flow derived from durable goods. The model shows that, in addition to the stock of durable goods, the household's time-varying utilization of durable goods also enters the pricing kernel. We develop an innovative method to identify the risk associated with the adjustable usage of durable goods using CO_2 emissions, exploiting the fact that a growing proportion of durable goods requires energy as a complementary input in their usage, and almost all types of energy consumption involve generating carbon dioxide (CO_2). We show that the utilization risk is an important dimension of consumption risk. Our model mitigates the joint risk premium and implied risk-free rate puzzle, and it exhibits a strong cross-sectional pricing power.

We highlight a risk that is associated with the time-varying utilization of durable goods. Many papers assume that utility is separable in durable and nondurable consumption or a constant constitution in the consumption bundle, and thus test the CCAPM only using data from nondurable consumption, despite that (1) these two components of consumption may not be separable (Eichenbaum and Hansen (1990)) and (2) durable goods have gained growing importance in households' consumption. Meanwhile, the durable goods literature, represented by Yogo (2006), has recognized the importance of durable goods but model the consumption solely by the stock of durable goods. In this paper, using a simple model, we illustrate that the risk associated with adjustable utilization of durable goods stock constitutes another important source of consumption risk. Following Yogo (2006) and many others, we adopt the Epstein and Zin (1991) recursive preferences in modeling the household's intertemporal utility, as doing so allows for the separation between relative risk

aversion (RRA) and elasticity of intertemporal substitution (EIS).[1] The representative household's intraperiod utility function, with a constant elasticity of substitution, is defined over the service flow of nondurable and durable consumption, where the service flow of durables is the product of the durable goods stock and an adjustable usage rate. The solution of the representative household's utility maximization problem implies a pricing kernel that depends on the nondurable consumption, the stock of durable goods, the durable usage rate, and the return to a wealth portfolio. When the elasticity of substitution between nondurable and durable service flow is higher than the EIS, both durables stock growth (as shown in Yogo (2006)) and the durables usage growth have a positive price of risk.

Measuring the time-varying service flow of durable goods is a challenge to all CCAPMs with durable goods. This is the reason why previous research assumes that the service flow is a constant fraction of durable goods stock. However, if the usage of a durable good involves consumption of a complementary good, the unobserved service flow can be indirectly measured by tracking the consumption of the complementary good. We exploit the fact that the constitution of durable goods has shifted toward the one that requires energy as a complementary product in its service flow generation process (e.g., cars, appliances, and computers) in the recent decades. Because almost all forms of energy consumption involve emissions of CO_2, emissions can serve as a convenient aggregator of the complementary energy consumption used to generate durables service flow. Therefore, we measure service flow of durable goods in terms of the amount of CO_2 emissions after controlling for emissions caused by nondurable consumption. Moreover, an emission-based measure of durables service flow allows us to isolate the growth of durable goods utilization, in addition to the durable stock growth, where the latter is commonly considered in the previous literature.

Consistent with our model's prediction, we find that the risk associated with adjustable usage of durable goods has a significant pricing power in U.S. portfolios. The pricing power is strong in the recent forty years when service flow of durable goods can be measured more accurately through the consumption of energy and aggregated via CO_2 emissions. The utilization factor yields a positive price of risk in a linearized four-factor model, where the other three factors studied in Yogo (2006),

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[1]Bansal and Yaron (2004), Pakos (2007), Gomes, Kogan, and Yogo (2009), Malloy, Moskowitz, and Vissing-Jørgensen (2009), Yang (2011), Cui (2012), and Ready (2012) also model consumption risk using a recursive utility. Hansen et al. (2007) provides a thorough discussion on the role these two preference parameters played in affecting the demand of investors and asset prices.

specifically the nondurable consumption growth, durables stock growth, and market return are present. This suggests that stocks with returns more correlated with the household's time-varying utilization of durable goods are awarded with higher compensation for bearing this undiversifiable risk. The model fits the cross-section of stock returns with an adjusted R^2 of 65%. The four-factor adjustable service flow model outperforms the CAPM and several CCAPMs, including a CCAPM with simple power utility function, a CCAPM with Epstein-Zin preferences, and Yogo's three-factor durable model. Inclusion of an utilization factor improves the adjusted R^2 by at least 11% relative to models listed above, suggesting that the utilization risk is indeed important. The positive and significant risk premium of the utilization risk is also verified in the Fama-MacBeth regressions.

In contrast to many CCAPMs with durable goods, where the cross-sectional and time-series pricing power often comes at a cost of high risk aversion, the proposed model with adjusted service flow can alleviate the joint risk premium and implied risk-free rate puzzle (Grossman and Shiller 1981; Shiller 1982; Mehra and Prescott 1985; Weil 1989). The estimation of conditional Euler equations results in a relative risk aversion of 10.5 when instruments are used. A less than one subjective discount rate (0.98) implies an annual real risk-free rate of 2%. The low RRA and the reasonable time preference arise from the feature that, whereas both changes in the nondurable consumption and the durable stock are smooth, the durable goods utilization growth is volatile and procyclical.

Our paper is related to several strands of literature. First, our analysis is part of the research that studies the role of durable goods in the constitution of consumption risk. It is related to Yogo (2006) as already discussed. Piazzesi, Schneider, and Tuzel (2007) study the role of housing goods and model the fluctuation in the relative share of housing consumption as a risk factor. Yang (2011) studies the long-run persistent risk in durable consumption and its asset pricing implications.[2] Our

Second, our approach to measuring the unobserved service flow contributes to the growing literature that uses alternative measures for household consumption. Savov (2011) uses garbage to measure nondurable consumption. Our paper differs from his as we focus on studying and measuring a new component of consumption risk which is associated with the utilization of durable goods. Da, Yang, and Yun (2014) use electricity usage as a proxy for service flow from household capital under the Beckerian framework of household production. They find that the growth of electricity usage, which is closely related to the growth of household capital, can help improve the cross-sectional pricing. Our paper considers the broad definition of durable goods that includes other important durable goods besides electric appliances (e.g. gas heater, cars), and focus on the changing utilization of the durable goods.

Third, our paper fits in the vast body of literature attempting to resolve the joint risk premium and implied risk-free rate puzzle. Other studies that consider alternative approaches to alleviate the problem include ones modeling the long-run risk of consumption (Bansal and Yaron 2004; Yang 2011[3]), ones measuring consumption growth over a longer horizon (Parker and Julliard 2005; Malloy, Moskowitz, and Vissing-Jørgensen 2009) or with alternative timing (Jagannathan and Wang 2007), and also ones that take into account the persistence of habit formation (Campbell and Cochrane 1999).

The rest of the paper is organized as follows. In Section 2, we start by presenting a consumption-based capital asset pricing model with adjustable service flow. We derive the pricing kernel, and demonstrate how the time-varying utilization of durable goods can affect asset prices. In Section 3, we outline how each component of consumption risk in the model is measured. Section 4 presents our empirical results. Section 5 concludes.

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[3]Zhou and Zhu (2012, 2015) show that an extension of the long-run risks model of Bansal and Yaron (2004) that allows two volatility components in the evolution of economic fundamentals can help to explain various puzzles including the equity premium puzzle.

2 A Consumption-Based Capital Asset Pricing Model with Time-varying Utilization of Durable Goods

To understand how the variation in the utilization of durable goods affects the stochastic discount factor (SDF) and explains the cross-section of assets returns, we model the household's consumption and portfolio choice problem with exogenous shocks in durables service flow. We layout the model in subsection 2.1 and present the pricing equations in subsection 2.2.

2.1 Model

We consider a two-period economy in which there are a large number of identical households. Based on Yogo (2006), in each period t, the representative household's utility arises from consumption of service flow of two goods: the nondurable good and the durable good. The household purchases C_t unit of the nondurable good and consumes it in the same period. Since consumption happens within the same period, service flow of the nondurable good can therefore be measured by C_t. On the other hand, the stock of the durable good lasts for more than one period. In each period, the household consumes the service flow, S_t, derived from the durable good stock (D_t).

The household's intraperiod utility, defined over the nondurable consumption and the service flow from the durable good stock, follows the constant elasticity of substitution (CES) form:

$$u(C_t, S_t) = ((1-\alpha)C_t^{(\epsilon-1)\epsilon} + \alpha S_t^{(\epsilon-1)\epsilon})^{\epsilon(\epsilon-1)}, \tag{1}$$

where $0 < \alpha < 1$ captures the utility share from the service flow of the durable good, and ϵ represents the intratemporal elasticity of substitution between the nondurable and the durable consumption.

We allow the utilization of the durable good to be adjustable. Specifically, we model the service flow of a durable good in period t as $S_t = U_t D_t$, where U_t captures the household's choice on how much service flow it consumes from the level of durable stock Dt. Relaxation of the constant utilization rate assumption allows us to better capture household's consumption and portfolio

choice.

The household's intertemporal utility is specified by the Epstein-Zin recursive function over the intraperiod utilities,

$$\mathbb{U}_t = \left\{ (1 - \beta)\mathfrak{u}(C_t, S_t)^{1-\frac{1}{\psi}} + \beta(E_t[\mathbb{U}_{t+1}^{1-\gamma}])^{\frac{1}{\theta}} \right\}^{\frac{1}{(1-\frac{1}{\psi})}}, \tag{2}$$

where $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$. Parameter $\beta \in (0,1)$ is the household's subjective discount factor; γ is the relative risk aversion (RRA); and, ψ is the elasticity of intertemporal substitution (EIS). The benefit of using this recursive function is that it nests many utility functions as its special cases, including the one that features additive separability between two types of consumption ($\epsilon = \psi$), the one that can be represented by an expected utility function ($\psi = 1/\gamma$), or the one that incorporates both features ($\psi = 1/\gamma = \epsilon$). In addition, the separation of the elasticity of intertemporal substitution and the risk aversion parameter is helpful in the context of this paper.

There are $N + 1$ tradable assets in the economy: one risk-free asset ($i = 0$) and N risky assets ($i = 1, \ldots, N$).[4] In each period t, the household invests $z_{i;t}$ unit of its discretionary wealth in asset i. The tradable asset i has a price of $P_{i;t}$ and a future payoff of $X_{i;t+1}$, with a gross return of $R_{i;t+1}$.

For the purpose of studying how the time-varying utilization of the durable good affects the SDF, we illustrate with a reduced-form model in which both components affecting the service flow of the durable good are assumed to exhibit time-variation but are set exogenously. That is, we assume that the household is endowed with a durable good stock of D_t and an utilization rate of U_t, in each period, at levels as if chosen optimally in an endogenous model. Whether the durable good stock and the utilization rate are endogenously chosen or exogenously given at optimal should not affect the SDF. The reduced-form model allows us to avoid the complexity associated with modeling the law of motion of the durable good stock,[5] the price of new durable stock, and the cost of converting the durable stock into service flow. We therefore choose to present the reduced-form model in the

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[4]The number of risky assets is irrelevant in our setup since the model is set in an endowment framework taking consumption as exogenous. In other words, the SDF does not depend on the assets traded.

[5]Past papers often assume the evolution of the durable good stock to follow the process $D_t = G_t + (1-\delta)D_{t-1}$, where G_t is new durable purchase in time t, and δ is the depreciation rate.

main section.[6]

The representative household faces two constraints in the reduced-form model: an intraperiod budget constraint that characterizes the tradeoff between consumption and asset investment in period $t = 0$,

$$W_t = C_t + \sum_{i=0}^{N} z_{i;t};$$

(3)

and, an intertemporal budget constraint that captures the evolution of the household's wealth over time,

$$W_{t+1} = \sum_{i=0}^{N} z_{i;t} R_{i;t+1}.$$

(4)

Given the household's current wealth W_t, the household makes the nondurable consumption and portfolio choice to maximize its recursive utility (2) subject to constraints (3) and (4).

2.2 Euler Equations

By solving the household's utility maximization problem, we arrive at a representation of the SDF,

$$M_{t+1} = \left[\beta (\frac{C_{t+1}}{C_t})^{-1=} \; (\frac{v(\frac{U_{t+1} D_{t+1}}{C_{t+1}})}{v(\frac{U_t D_t}{C_t})})^{1= \, -1=} \; (R^*_{W;t+1})^{1-1=} \right],$$

(5)

where,

$$v(\frac{U_t D_t}{C_t}) = \left[1 - \alpha + \alpha (\frac{U_t D_t}{C_t})^{1-1=} \right]^{1=(1-1=)},$$

(6)

with $R^*_{W;t+1}$ being the return on the optimal wealth portfolio. The household's first-order conditions imply the Euler equations:

$$E_t [M_{t+1} R_{i;t+1}] = 1.$$

(7)

The functional form of the SDF suggests that, as long as the utility is not additively separable in nondurable and durable consumption (i.e., $\epsilon = \psi$), the relative consumption of service flow from

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[6]We show in Appendix A that the SDF derived under two alternative settings, where we let either the durable stock or the utilization rate be endogenously determined, is the same as the one derived in the reduced-form model. The additional degree of endogeneity only leads to additional Euler equations that set the ratio of marginal utilities of goods equal the price ratio. We could not get a closed-form solution for Euler equations under the fully endogenous model. For the purpose of understanding the relation of consumption risks and asset pricing in equilibrium, the Euler equation is sufficient. We will leave estimation of the fully endogenous model to future work.

durable goods to nondurable consumption also affects the household's marginal utility, in addition to the nondurable consumption growth and the return on the wealth portfolio. Time-varying utilization rate matters in our model: it affects the service flow generated from the durable good. When $\theta > 0$ (i.e., $\gamma < \frac{1}{\ }$) and the elasticity of substitution between two types of consumption exceeds the elasticity of intertemporal substitution (i.e., $\epsilon > \psi$), an increase in the ratio of service flow ($\frac{U_t D_t}{C_t}$) decreases the household's marginal utility. Yogo (2006) shows that the procyclical D_t/C_t ratio can induce countercyclicality in marginal utility. The SDF derived from our model, featuring time-varying utilization of durable goods, suggests that a procyclical utilization of the durable good could further magnify the countercyclicality in marginal utility.

We can approximate the SDF in the following form [7]:

$$\frac{M_{t+1}}{E\left[M_{t+1}\right]} = 1 - b'(f_{t+1} - \mu_f), \tag{8}$$

where $f_t = (\Delta c_{t+1}, \Delta d_{t+1}, \Delta u_{t+1}, r_{W;t+1})'$, $\mu_f = (E[\Delta c_{t+1}], E[\Delta d_{t+1}], E[\Delta u_{t+1}], E[r_{W;t+1}])'$, and the prices of risk $b = (b_1, b_2, b_3, b_4)'$ can be expressed as functions of preference parameters:

$$b_1 = \theta(\frac{1}{\psi} + \alpha(\frac{1}{\epsilon} - \frac{1}{\psi})), \tag{9}$$

$$b_2 = \theta\alpha(\frac{1}{\psi} - \frac{1}{\epsilon}), \tag{10}$$

$$b_3 = \theta\alpha(\frac{1}{\psi} - \frac{1}{\epsilon}), \tag{11}$$

$$b_4 = (1 - \theta). \tag{12}$$

Δc_{t+1}, Δd_{t+1}, and Δu_{t+1} are log growth rates of the nondurable consumption, the durable good stock, and the utilization rate of the durable good stock; and, $r_{W;t+1}$ is the log return on the optimal wealth portfolio.

Under such approximation, the Euler equation implies a linear factor model for assets' excess

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returns (Equation (13)).

$$E\left[R_{i;t+1} - R_{0;t+1}\right] = b_1 Cov(\Delta c_{t+1}, R_{i;t+1} - R_{0;t+1}) + b_2 Cov(\Delta d_{t+1}, R_{i;t+1} - R_{0;t+1}) \quad (13)$$
$$+ b_3 Cov(\Delta u_{t+1}, R_{i;t+1} - R_{0;t+1}) + b_4 Cov(r_{W;t+1}, R_{i;t+1} - R_{0;t+1}).$$

Notice that the price of risk is the same for the durable stock growth and the growth of the durable good utilization. This is not surprising given that they jointly enter the household's utility function through the service flow of the durable good. Their prices of risk are both positive when the elasticity of substitution between service flow from the durable and the nondurable good is sufficiently large relative to the EIS (i.e., $\epsilon > \psi$).

3 Data

3.1 The Nondurable Consumption Risk and the Durable Stock Risk

We follow convention on measuring the nondurable consumption and durable goods stock. Nondurables consumption is measured as the real per capita personal expenditure on nondurable goods and services. The data is from the U.S. National Income and Product Accounts (NIPA) at the Bureau of Economic Analysis. NIPA defines nondurable goods and services as commodities consumed either at the time of purchase or with a life span of less than three years. The annual real expenditure series, measured in 2005 constant dollar, is computed using nominal expenditures on nondurable goods and services and their corresponding price indices.[8] Since we work with data in annual frequency, no seasonal adjustment is needed for any of the consumption data (Ferson and Harvey 1993).

Durable goods, in NIPA's definition, are commodities that can be stored or inventoried and have an average service life of at least three years. These include items such as motor vehicles, furniture, appliances, jewelry, watches, and recreational goods. While the service flow of durable goods is difficult to measure, NIPA provides data on the year-end chained quantity index of real durables

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[8]NIPA tables only provide price indices on nondurable goods and services separately. We derive the corresponding price index for the sum of nondurable goods and services using the nominal expenditure and the quantity index of these two categories of expenditure (NIPA Tables 2.3.5 and 2.3.3), following Yogo (2006).

net stock and its price index. We construct an annual series of real net stock of durable goods measured in 2005 constant dollar using the chained quantity index and the price index following Yogo (2006).[9]

Both nondurables expenditure and durables stock are scaled by population using population data from the U.S. Census Bureau. We use the average population in year t and year

expenditures constructed following an approach described in Gomes, Kogan, and Yogo (2009).

3.2 The Durable Good Utilization Risk

Our objective is to keep track of how households use their durable goods from time to time. While records of the households' net stock of durable goods give information about how much durable goods the household owns, they provide little information on how the household uses them differently over time. Usage rate and stock value can have different time series variation. Take cars as an example, although the market value of cars can be relatively smooth across time, the households may choose to drive more or less at different times. The same logic applies to many other durable goods.

We empirically identify the time-variation in durable goods usage rate using an indirect approach: if a durable good uses a complementary product to generate its service flow, then measuring the consumption of the complementary product offers an indirect way to measure the service flow from that durable good, which allows us to back out the time-varying utilization rate. When the fraction of durable goods that requires the complementary product in their service flow generation is sufficiently large, consumption of these complementary products can be useful in capturing the overall service flow from durable goods. The observation in recent decades, of a growing share of durable goods that requires energy as a complementary input in their usage, therefore opens a new window on the measurement of durables service flow. Durable goods that require energy as an input include motor vehicles, household appliances, computers, sports and recreational vehicles, therapeutic appliances, telephone and facsimile equipment, etc. In contrast, goods that are relatively self-contained in the service flow generation process include furniture, household equipment, books, jewelry and watches, luggage, and others. The solid line in Panel B of Figure 1 plots the stock ratio between durable goods that are energy dependent to the ones that are not. Households' holdings of durables stock is shifting toward energy-dependent durable goods in the latter half of the sample: the ratio exceeds one after 1970 and continues to increase; the average ratio is 55.6% higher in the post-1970 sample relative to the earlier sample, and the difference is significant with a t-statistic of 11.7.[12] Behind the growth in share is the fast growth in the usage of

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[12]The huge dip and acceleration in the fortieth is caused by the World War II rather than the fundamental changes

cars, light trucks (vans, minivans, sports utility vehicles, etc.), and appliances. Panel B of Figure 1 provides a graphical demonstration. These changes are not only associated with the advance in technology that made the relative price of these durable goods less expensive but also related to changes in consumption style. For example, household consumption has changed toward having more traveling, leisure activities, and services, all of which involve intensive usage of transportation. At the same time, the increase in women's participation in the labor force turns home production into consumption offered by specialists. They contribute to the growth of cars and light truck usage and thus the overall growth in the importance of energy-dependent durable goods.

However, the utilization of these durable goods involves consumption of many types of energy inputs that take different forms and units. For example, gasoline is the predominant energy input for driving motor vehicles; most home appliances require electricity; some other durable goods use natural gas or even heating oil. CO_2 emissions offers a way to conveniently and comprehensively summarize energy consumption in the United States: almost all types of energy used by durable goods lead to CO_2 emissions, either at the time of consumption (e.g., gasoline) or at the time of energy production (e.g., electricity).[13] In addition, relative to the expenditure-based energy consumption, emissions-based aggregation is less likely to be subject to fluctuations of energy prices which are known to be correlated with many macroeconomic factors. Therefore, by using CO_2 emissions data, we propose a methodology to identify the unobserved service flow and the time-varying utilization of durable goods.

The CO_2 emissions data are from the Carbon Dioxide Information Analysis Center (CDIAC) in the Oak Ridge National Laboratory (ORNL). The dataset has annual CO_2 emissions from fossil fuel combustion and cement production,[14] measured in thousand metric tons of carbon.

Time series of CO_2 emissions are constructed based on historical records of energy consumption. Emissions are computed by applying CO_2 emissions conversion coefficients to energy consumption series.[15] Specifically, CO_2 emissions of fuel type i are estimated as the product of three terms: the

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in consumption patterns.

[13]Energy-related CO_2 emissions account for about 98% of the U.S. CO_2 emissions.

[14]The vast majority of CO_2 emissions comes from combustion of fossil fuel. A very few number of emissions come from the nonfuel use of fossil fuels, electricity generation using geothermal energy, and nonbiomass waste.

[15]Details on the contents and processing of the historical energy statistics provided in Marland and Rotty (1984) and Andres et al. (1999). The 1950 to 2011 CO_2 emission estimates are derived from energy statistics published by the United Nations.

quantity consumed of fuel type i, the carbon content of fuel type i, and the fraction of the carbon content that gets oxidized.[16] Quantities of fuel consumption are controlled for changes in the form of fuel, imports and exports of fuel, and changes in fuel stocks. They provide good estimates for the amount of fuels that generate CO_2 emissions, as a result of energy consumption.

The total CO_2 emissions in our dataset is an aggregation of CO_2 emissions generated from the combustion of solid fuel, liquid fuel, gas fuel, and cement productiom. Solid fuel refers to various types of solid materials, mainly charcoal and coal, that are used to produce energy. The primary use of solid fuel is in electricity generation: as of 2011, 92% of the solid fuel was used in electricity generation, which in total contributes to nearly half of the total electricity generation in United States. The importance of solid fuel in electricity generation is even more pronounced in earlier years before the introduction of non-fuel-based inputs. Liquid fuel includes gasoline, distillates, kerosene, compressed natural gas, and liquid petroleum gas. The use of liquid fuel is dominated by transportation: the 2011 data shows that 71% of the total consumption of liquid fuel in the United States was for transportation use. Liquid fuel is also used in heating homes and industrial production. Gas fuel refers to natural gas. Natural gas can be used to produce glass, paper, clothing, and other consumption goods, but its main usage is in electric power generation and house heating. As of 2011, natural gas was still the second most important source for electricity generation after coal. In addition, more than half of the homes in the United States use natural gas as their main heating fuel. Natural gas is also used to fuel stoves, water heaters, clothes dryers, and other household appliances. CO_2 can also be released in cement manufacture when calcium carbonate is heated during the manufacturing of cement. However, it accounts for a very small fraction of total CO_2 emissions.[17] The composition of CO_2 emissions has changed over the years reflecting changes in consumption: in 1930, fractions of emissions from combustion of liquid, gas, solid fuel, and cement production are 21%, 5.7%, 73.5%, and 0.8% respectively; in 2011, these numbers are 39.7%, 24.4%, 35.2%, and 0.6%. The shift from coal-based emissions to liquid- and gas-based emissions mirrors the change in technology of electricity generation, and the increase in

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[16]The CDIAC assumes the carbon content and oxidization ratios to be time invariant. Whereas the carbon content of fuel has not varied considerably since the nineteenth century, the fraction that can be oxidized varies because of improvements in combustion efficiencies and non-fuel usage. Although these non-fuel uses have increased over time, so have combustion efficiencies. These two effects counter each other, and therefore the product can be assumed constant.

[17]Our results remain robust if we use total CO_2 emissions excluding emissions caused by cement manufacture.

the usage of automobiles and household appliances.

We propose a methodology to identify the service flow of durable goods and the risk associated with the time-varying utilization of durable goods, denoted by S_t and Δu_t, respectively in the model. We recognize the fact that emissions could arise not only from durable consumption but also from nondurable consumption.[18]

the emissions from durables service flow essentially assumes independence between emissions from two sources of consumption. Our estimated emissions from durables service flow is therefore a conservative measure as nondurables and durables consumption exhibit comovement to some degree in reality.

Second, the conversion rate of durables service flow to emissions (b_s in Equation (14)) is assumed to be the same across all durable goods. In reality, the true conversion rate is in fact zero for durables that require no energy consumption in their service flow generation (e.g., furniture, books, and luggage) and positive for the ones that use energy (e.g., cars, appliances, and computer). However, we can think of b_s as an average conversion rate. As long as the rate is relatively stable over time, it should not affect our estimation of the utilization risk in the second regression. This suggests that our identification strategy works better in the period when the fraction of energy dependent durable goods is high and the composition is relatively stable. This is one of the reasons why we focus on the sample period of 1970–2010.

Third, the conversion rate between consumption and emissions is assumed to be time-invariant. Technology improvement and the increasing public awareness towards environmental protection has definitely affected CO_2 emissions. However, technology is more likely to have a lower frequency and is less likely to exhibit large volatility at annual frequency. Thus, the slow-moving variation associated with technology changes should have a limited impact on the measurement of the utilization risk. To be cautious, in the robustness analyses, we remove time trend from emissions and the results remain similar.

Fourth, Equation (14) assumes that consumption is the only source for CO_2 emissions. In reality, emissions can come from production, investment, or simply heating (cooling) caused by extreme weather conditions. In Section 4.3, we show that results remain robust after we control for these potential sources of emissions.

Lastly, using a non-expenditure-based consumption measure raises the issue that the weights used in the aggregation may not match with consumption goods' importance in the household's preference function. Using emissions as a proxy for service flow essentially assumes that usage of durable goods that emit more CO_2 play more important roles in the constitution of the adjustable

utilization risk. We admit that this could potentially affect the quality of the utilization rate measurement. However, our contribution stands more on distinguishing the time variation of durables stock and service flow.

3.3 Asset Market Data

Stock market data used in this paper consist of the U.S. stock market index and a number of U.S. stock portfolios. Return data of the value-weighted U.S. stock market are available from the Center for Research in Security Prices for the period of 1930–2012. For U.S. stock portfolios, we use the Fama-French three factors, 25 portfolios sorted on size and book-to-market, 10 portfolios sorted on earnings/price, 10 portfolios sorted on cashflow/price, and 10 portfolios sorted on dividend yield. All portfolios are downloaded from Kenneth French's data library. Real excess returns of these portfolios are computed using nominal returns, one-month U.S. Treasury bills as the risk-free rate, and the personal consumption price index from NIPA as the inflation deflator.[19] We also use dividend-price ratio, long-short yield spreads, size spread, and value spread as instruments in the conditional estimation of Euler equations. We follow Appendix A(II) in Yogo (2006) for the construction of these instruments.

3.4 Data Properties

Table 1 presents descriptive statistics of the three types of consumption risk (the nondurable consumption growth, the durables stock growth, and the utilization risk), their pairwise correlations, and their correlations with alternative nondurable consumption growth measures and the real market excess return.

[Insert Table 1 here]

We highlight a number of observations: first, the mean of the durables stock growth is 3.9% over the period of 1970–2010, which is higher than the mean of the nondurable consumption growth, reflecting its growing importance in recent decades. Second, the U-factor is slightly more volatile than the nondurable expenditure growth and the durables stock growth: the standard deviation

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[19]Item 2 in NIPA Table 1.1.4.

of the U-factor is 2.2% over the period of 1970–2010, relative to 1.9% and 1.5% for durables and nondurables. Third, the U-factor is less autocorrelated than the consumption growth series. Specifically, it has a first-order autocorrelation coefficient of 12.8% in contrast to 78.4% associated with the durable stock growth.[20] The contrast in persistence is in line with our intuition that the household's usage of durable goods can vary from time to time while the change in the stock value is relatively smooth. We plot the time series of these three factors in Figure 2. Lastly, both the U-factor and consumption growths have positive correlations with the real market return. The U-factor has a correlation of 51.4% with the market return, compared to 17.2% for the durables stock growth.

[Insert Figure 2 here]

4   Empirical Results

The empirical tests are conducted in twofolds. We examine the cross-sectional implications of the model by performing GMM estimations on the log-linearized pricing equations. We compare our model's performance against other models, including the CAPM, the Fama-French three-factor model, the CCAPM with power utility, the CCAPM with Epstein-Zin preferences, and Yogo's durable good consumption model. We also supplement tests of the linear factor model using Fama-MacBeth regressions. We then estimate preference parameters through the unconditional and conditional moment restrictions imposed by the Euler equations. In the robustness analysis section, we estimate U-factors after controlling for alternative sources of emissions, including production, investment, the weather effect, and technology changes. The default set of testing assets is the 25 Fama-French portfolios formed on size and book-to-market.[21]

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[20]The AR(1) coefficient for the nondurables consumption growth series is 45.6%. Working (1960) and Breeden, Gibbons, and Litzenberger (1989) suggest that this is likely to be due to time aggregation and their rigidity in adjustment.

[21]Due to the limited length of the data, we use the 25 Fama-French portfolios in the GMM estimation of the linear factor model and the unconditional Euler equations. We use the Fama-French three factors in estimating the conditional Euler equations because they can capture the common variation in returns across the 25 Fama-French portfolios while keeping the number of assets small. We include an additional 30 portfolios in the Fama-MacBeth regressions analysis.

4.1 Cross-Sectional Tests of the Linear Factor Model

The unconditional Euler equations stated in Equation 7 imply that excess returns on assets $i = \{1, \ldots, N\}$ over the risk-free asset $i = 0$, denoted by $R^e_{i;t+1} = R_{i;t+1} - R_{0;t+1}$, must satisfy the following pricing equations:

$$E_t\left[M_{t+1}R^e_{i;t+1}\right] = 0, \tag{17}$$

where M_{t+1} is the SDF defined in Equation (6). Combining with the log-linear approximation of the SDF in Equation (8), the pricing equations can be approximated by a linear factor model:

$$E[R^e_{i;t+1}(1 - b'(f_t - \mu_f))] = 0. \tag{18}$$

Following Cochrane (2005), we use the following moment conditions,

$$E\left[\begin{array}{c} R^e_{i;t+1} - R^e_{i;t+1}b'(f_t - \mu_f) \\ f_t - \mu_f \end{array}\right] = 0, \tag{19}$$

where the prices of risk b and the factor mean μ_f are unknown coefficients to be estimated. We implement a two-step GMM estimation, using real excess returns of the 25 Fama-French portfolios. Details on the estimation can be found in Appendix C.1.

Panel A of Table 2 reports the estimated prices of risk under the four-factor durable model. The U-factor is priced in the cross-section of stock returns: it has an estimated price of risk of 42.48 with a t-statistics of 5.38. The significant price of risk on the U-factor suggests that variations in the adjustable service flow of durable goods is a valid source of risk in pricing U.S. stock portfolios. The nondurable consumption growth and the durables stock growth also carry positive and statistically significant prices of risk: the price of risk for nondurable consumption growth is 24.22 (t-statistics=2.70), and the price of risk for durable stock growth is 41.63 (t-statistics=9.06). Further, the estimated price of risk on the U-factor is very similar to the one estimated for the durable goods stock growth. In fact, a Wald test with the hypothesis of these two prices of risk being equal cannot be rejected. This finding is consistent with the model's prediction. The price of risk for the return on the wealth portfolio, proxied by the market return following

Epstein and Zin (1991), is negative and significant. We do not have a good explanation for it, but the negative significant risk price on the market return is found under all consumption-based models (Panel B Table 2), including the simple CCAPM with power utility (CCAPM), the CCAPM with Epstein-Zin preferences (CCAPM-EZ), and Yogo's three-factor durable good model.

To assess the goodness-of-fit, we compute the pricing error measured by root-mean-squared error (RMSE) and the adjusted R^2. The RMSE for the four-factor model is 1.75%, and the adjusted R^2 is 65%. The J-test fails to reject the model at the 5% significance level, suggesting that our model prices the 25 Fama-French portfolios.

[Insert Table 2 here]

Preference parameters can be backed out from the estimates of prices of risk given the relationship in Equation (9) to (12). The subjective discount rate β does not enter the linear factor model and thus cannot be implied by the prices of risk. Following Yogo (2006), we set ϵ to be 0.79 in order to overcome the issue of the intratemproal elasticity of substitution parameter ϵ and the share parameter α being not separately identifiable. Estimates for the remaining preference parameters are: a RRA (γ) of 61.76, an EIS (ψ) of 0.08, and a share parameter (α) of 0.70. All of them are statistically significant at over 99%. The close to one EIS is similar to the ones found in other CCAPMs with durable goods. The large α is consistent with the empirical observation of an increasing importance of durable consumption in recent decades.

Our four-factor model featuring time-varying utilization of durable goods can explain the cross-section of stock returns better than several comparable models, including the CAPM, the Fama-French three-factor model, and three CCAPMs.[22] The results are shown in columns (b) to (f) in Panel B of Table 2. We find that, although the standard CAPM delivers a positive significant price of risk, its poor pricing performance is reflected in a large RMSE of 3.41% and a negative adjusted R^2 of -0.35.[23] The Fama-French three-factor model, on the other hand, has a superior pricing performance: it generates a small RMSE of 1.39 and a large R^2 of 0.78. The CCAPM with

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[22]We conduct the comparison over the same sample period of 1970-2010 for consistency as Ferson and Merrick (1987) suggest that model parameters may shift across time periods.

[23]Adjusted R^2 is computed as one minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of the realized average excess returns, following Campbell and Vuolteenaho (2004). It can take a negative value when a model's pricing errors are worse in the way that its cross-sectional variance is greater than the cross-sectional variance of the realized average excess returns.

power utility has the poorest goodness-of-fit among all consumption-based models: whereas the price of risk on nondurables is large and significant, the pricing error is 2.47%, and the adjusted R^2 is only 29%. The two-factor CCAPM with Epstein-Zin preferences reduces the RMSE to 1.99% and almost doubles the adjusted R^2 to 54%. The price of risk on nondurables is at 140 under the CCAPM with Epstein-Zin preference. This large value may be explained by low volatility of the nondurable consumption growth, consistent with the argument in Yogo (2006) and Savov (2011). Under Yogo's three-factor durable good model, prices of risk for nondurables and durables are both positive and significant. However, the inclusion of the durables stock growth does not seem to improve the pricing performance by much from the CCAPM-EZ. We find that our four-factor model outperforms all consumption-based models in terms of goodness-of-fit. The inclusion of the U-factor helps to reduce the pricing error by a magnitude of 0.24% and improve the adjusted R^2 by 11% from Yogo's model. A similar level of pricing performance can also be achieved if we collapse the U-factor and the durables stock growth factor into a durables service flow factor — the improvement is not purely coming from having an additional factor in the pricing regression.

We back out the implied RRAs for the consumption-based models. The CCAPM with power utility delivers a RRA of 89. On the other hand, the two-factor CCAPM with Epstein-Zin preferences yields a RRA of 137. Further, Yogo's durable good model helps to reduce the estimate to 118. Among all models, our four-factor model featuring time-varying utilization of durable goods yields the lowest RRA of 62.

Past studies on CCAPMs have proposed a number of alternative measures for nondurable consumption growth. Table 3 reports prices of risk, implied preference parameters, and goodness-of-fit measures of our four-factor model using alternative measures of nondurable consumption: the ultimate consumption risk factor used in Parker and Julliard (2005) (PJ) and the fourth-quarter-to-fourth-quarter consumption growth proposed by Jagannathan and Wang (2007) (Q4-Q4). Inspired by Gomes, Kogan, and Yogo (2009), we also construct a new set of consumption risk proxies by adjusting the benchmark measures to count housing as part of durable goods instead of services and to remove expenditures on utilities from nondurables expenditures (NDexUH).[24] Results show

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[24]Specifically, nondurable and service consumption is measured as the chain-weighted sum of real personal consumption expenditures on nondurable goods, plus services, and minus expenditures on energy products and electricity, minus housing services. Stock of durables is the net stock of consumer durable goods plus the net stock of private

that price of risk of the U-factor is positive and significant across alternative nondurable consumption measures. Further, consistent with the model's prediction, prices of risk on the durable stock and the U-factor are similar: Wald tests cannot reject the hypothesis that two prices of risk are the same. The implied RRA (γ) are positive and significant, with values of 65 (PJ), 78 (Q4-Q4), and 87 (NDexUH). Goodness-of-fit, measured by the RMSEs and the adjusted R^2s, is also very comparable.

[Insert Table 3 here]

Given the limited length of our sample, we are restricted in the number of testing assets. We expand our cross-sectional study to a larger set of testing assets by implementing two-pass Fama-MacBeth (1973) regressions.

The pricing equation can be recast to the form shown in Equation (20) where assets' excess returns are expressed as the product of risk loadings (β) and factor risk premia (λ). Factor risk premia can be rewritten in terms of the prices of risk in Equation (13) scaled by factor' covariance matrix.[25] The larger set of testing assets contains the 25 Fama-French size and book-to-market sorted portfolios and 30 additional portfolios: 10 portfolios formed on earnings/price, 10 portfolios formed on cashflow/price, and 10 portfolios formed on dividend yield. In addition to our four-factor model, we also present factor risk premia estimated under the CCAPM-EZ and Yogo's durable good model, and a special case of a two-factor model with the U-factor and the market factor.[26]

$$E[R^e_{i;t+1}] = \lambda' \beta_i \tag{20}$$

Panel A of Table 4 presents factor risk premia estimated using 55 portfolios over the sample period of 1970–2010. Consistent with the GMM estimations, the U-factor has a positive and significant factor risk premium under our four-factor model. The factor risk premium is 2.55 with a t-statistic of 2.55. Although nondurable and durable goods factors are weakly significant under Yogo's three-factor model (t-statistics are 1.95 and 1.72, respectively), they do not have significant factor risk

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residential fixed assets (from NIPA Fixed Assets Tables).

[25]Let $_f = E[f_t]$, $\Sigma_{ff} = E[(f_t - _f)(f_t - _f)']$, and $\Sigma_{fi} = E[(f_t _f)(R)it - R_{0t})]$ and b be the risk prices such that $E[R_{it} - R_{0t}] = b'\Sigma f_{it}$, then $= \Sigma_{ff}b$.

[26]The two-factor model with U-factor and the market return does not have a theoretical foundation. The objective is simply to look at the empirical pricing power of the utilization risk when it is used alone with the market as a control.

premia under our four-factor model when the U-factor is included. Among these three consumption-based models, our four-factor model has the best goodness-of-fit: it has the highest adjusted R^2 of 55% (relative to the 53% of Yogo's and 49% of the CCAPM-EZ) and the lowest RMSE of 1.30% (relative to the 1.44% under Yogo's and the 1.44% of the CCAPM-EZ). Notably, the U-factor also has a significant factor risk premium when the market factor is controlled for. The factor risk premium is 4.07 with a t-statistic of 2.12, and an adjusted R^2 is 43%.

[Insert Table 4 here]

The first row in Panel B of Table 4 present factor risk premia estimated using an earlier subsample of 1930–1969. The U-factor has an insignificant factor risk premium for the earlier subsample (t-statistics=0.63). The adjusted R^2 of the two-factor model is 39% over the earlier sample, which is significantly lower than the 55% using the more recent sample. Further, longer samples containing the early periods also yield weak pricing results using the 25 portfolios (row 2) or the 55 portfolios (row 3). These results illustrate that changes in consumption of durable goods toward the ones that require energy as a complementary input play a key role in our identification of the durables utilization risk from CO_2 emissions.

Two innovative measures of consumption have been introduced recently. One is Savov (2001)'s garbage measure for nondurable consumption, and the other is Da, Yang and Yun (2014)'s electricity usage measure for household production. In this paper, we use CO_2 to measure utilization of durable goods. CO_2 serves as a better measure for capturing such risk: usage of most durable goods generates little solid waste; and, electricity accounts for the usage of some household durable goods while excluding many elements such as cars usage and gas heating. Pricing results also confirm this. Panel C of Table 4 presents the factor risk premia of the U-factor in which the U-factor is constructed using garbage/electricity in place for CO_2: garbage-based U-factor has a negative factor risk premia; while the factor risk premia on the electricity-based U-factor is positive but insignificant.

4.2 Estimates of Preference Parameters

Preference parameters can be estimated directly using the unconditional and conditional moment conditions implied by the Euler equations of our adjustable service flow model. We follow Hansen and Singleton (1982) in the construction and the testing of moment restrictions as in Equations (21) and (22). M_{t+1} is the SDF as defined in Equation (6); $R_{i;t+1}$ represents the return of asset i ($i = 1, \ldots, N$) over the period $t + 1$; $R_{0;t+1}$ is the return of the risk-free asset; and I_t is an $I \times 1$ vector of instrumental variables known at time t.[27] There are $(N + 1)I$ moment conditions in total.

$$\mathbf{E}_t\left[(M_{t+1}R_{0;t+1} - 1)I_t\right] = 0, \tag{21}$$

$$\mathbf{E}_t\left[(M_{t+1}(R_{i;t+1} - R_{0;t+1})I_t\right] = 0, \quad i = 1, \ldots, N. \tag{22}$$

A challenge associated with the estimation of Euler equations is that we cannot identify the level of the utilization rate U_t — our identification only gives the log growth of the utilization risk. Although it is not a problem for the linear factor model as the SDF is approximated as a linear combination of the log growth rates, it is indeed a problem in the Euler equations estimation. However, our identification allows us to obtain a proxy of the durable goods service flow measured in units of CO_2 emissions, represented by $\hat{a}_1 + \hat{e}_t$ in Equation (15). Therefore, in the estimation of the Euler equations, we work with service flow of durable goods ($S_t = U_t D_t$), rather than with the level of durable goods stock (D_t) and the utilization rate (U_t) separately. The difference in units remains an issue: the nondurables consumption is measured in constant-dollar per capita, whereas the durables service flow is measured in metric tons of CO_S per capita.

To account for differences in units, we modify the utility function by including an additional preference parameter on the relative importance of nondurable and durable goods. Specifically, we replace the weight of the nondurable consumption $(1 - \alpha)$ and the weight of the service flow from durable goods (α) in the intraperiod utility function by two separate parameters α_1 and α_2. The modified functional form of the SDF is presented in Equations (23) and (24). By relaxing the restriction of the sum of weights equal one, we let the two new parameters to absorb the differences

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[27]In the unconditional moments, I has an dimension of one and I_t is a scalar takes the value of one.

in units. However, because the conversion between expenditures and emissions is unknown, the values of these two parameters carry little economic meaning.

$$M_{t+1} = \left[\beta(\frac{C_{t+1}}{C_t})^{-1=} \; (\frac{\upsilon(\frac{S_{t+1}}{C_{t+1}})}{\upsilon(\frac{S_t}{C_t})})^{1= -1=} \; (R^*_{W;t+1})^{1-1=}\right] \qquad (23)$$

$$\text{where } \upsilon(\frac{S_t}{C_t}) = \left[\alpha_1 + \alpha_2(\frac{S_t}{C_t})^{1-1=}\right]^{1=(1-1=)}$$

To estimate the six preference parameters (ψ, γ, β, ϵ, α_1, and α_2), we use the one-month Treasury bill rate for $R_{0;t}$, the 25 Fama-French portfolios for $R_{i;t+1}$ in the case of the unconditional model, and the Fama-French (1993) three factors in the case of the conditional model. Instruments for the conditional model include second lags of nondurable consumption growth, durables service flow growth, dividend-price ratio, long-short yield spread, size spread, value spread, and a vector of ones.[28] With our selection of testing assets and instruments, we have an overidentified GMM system in either the unconditional or the conditional test. Parameters are estimated using a two-step efficient GMM.

[Insert Table 5 here]

Panel A of Table 5 presents estimates for the preference parameters under the unconditional model. The estimated EIS is 0.06 (t-statistic $= 5.05$). The magnitude is close to zero, which is similar to the estimates in Yogo (2006) (0.024) and in Da, Yang, and Yun (2013) (0.016). The estimate for the RRA is 57 (t-statistic $= 4.32$), similar to the implied value from the linear factor model. The subjective discount rate β is 0.91, which can be translated into an annualized real risk-free rate of close to 10%. The elasticity of substitution between two types of service flows (ϵ) is 2.29 but insignificant. Overall, parameters estimated using the unconditional moment conditions are similar to those implied by the linear factor model.

Panel B of Table 5 presents preference parameters estimated using conditional moment conditions with instruments. The estimation produces a RRA of 10.51 (t-statistics $= 5.88$), and a subjective discount rate β of 0.98 (t-statistics $= 4.36$), which translates into a real risk-free rate of 2%. Being able to obtain a relatively low RRA and a close to, but less than, one subjective discount rate

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[28]The choice of instrument is similar to Yogo (2006) and Da, Yang, and Yun (2014).

suggests an alleviation of the joint risk premium and implied risk-free rate puzzle.

4.3 Robustness Analyses

The benchmark identification for the utilization risk is based on the assumption that CO_2 emissions arise from nondurable and durable consumption only. However, in reality, CO_2 emissions may be influenced by other factors. In this section, we conduct robustness tests to show that our results remain even after controlling for these alternative sources of CO_2 emissions.

First, it is possible that some CO_2 emissions are generated as a result of investment and production. According to the U.S. Environmental Protection Agency, in 2011, various industrial processes accounted for about 14% of total U.S. CO_2 emissions; 20% of electricity use, which plays an important role in CO_2 emissions, was for industrial use. To control for emissions from production and investment, we add the gross domestic product (GDP) and private fixed investment, each separately, as an additional source for CO_2 emissions in the estimation of durables service flow.

$$CO_{2;t} = b_c C_t + b_s S_t + b_a A_t \tag{24}$$

Specifically, we first regress CO_2 emissions on nondurable consumption C_t and the additional factor A_t. The sum of the constant and residuals are taken as a proxy for durables service flow. The utilization risk, a.k.a. the U-factor, is measured by the residuals from regressing the log-growth of the estimated emissions from durables service flow on the log-growth of durables stock. We conduct the two-pass Fama-MacBeth regressions using 55 portfolios. The results are reported in the first two rows of Table 6, Panel A. In both cases, the factor risk premium on the U-factor is positive and significant. In addition, the RMSEs and adjusted R^2s are about the same, compared with those in the baseline model.

Second, weather may affect CO_2 emissions because a substantial amount of household energy is used in cooling and heating. Since emissions are measured in annual frequency, the within-a-year weather fluctuations should have little impact on the estimation of the U-factor. However, cross-year variation in weather may have an impact on the amount of emissions. To make sure our results are not driven by the weather effect, we include the Energy Degree Day (EDD) variable in the first-

stage regression. EDD is a measurement that quantifies demand for energy needed in heating and cooling. Results, in the third row of Table 6, Panel A, indicate that pricing performance of our four-factor model is not affected after controlling for the weather effect. The factor risk premium on the U-factor is positive and significant, and the goodness-of-fit measures remains almost unchanged.

[Insert Table 6 here]

Third, conservation and technology changes are expected to have an impact on CO_2 emissions. The public's awareness of environmental protection and improvement of fuel efficiency can lead to a reduction in CO_2 emissions, especially in the more recent decades. However, changes in technology and actions taken toward preserving conservation are mostly long-run phenomenons and have impacts on the level of CO_2 emissions in a gradual manner. In contrast, the risk associated with the usage of durable goods has a higher frequency. As a result, conservation and technology are less likely to add much noise in the measurement of utilization risk. However, to be cautious, we adjust the U-factor by first detrending CO_2 emissions. This approach provides a more conservative remedy, as the inclusion of the time-trend variable removes the slow-moving effect on emissions, including, but not limited to, the ones caused by conservation and technology changes. Results are shown in the last row of Table 6, Panel A. The U-factor continues to perform well even after controlling for the slow-moving time effect.

We also find robust results in the GMM estimations of the linear factor model, as in Table 7. Although magnitudes of prices or risk and preference parameters are different across various adjusted U-factors, the key findings are the same. First, the U-factor has a positive and significant price of risk in all cases. Second, consistent with the theoretical prediction, prices of risk on the U-factor and durable goods stock are fairly similar — Wald tests of equal risk prices are never rejected. Third, all preference parameters are statistically significant. The RRA estimates range from 60 to 108. EIS estimates are positive with values close to zero. The relative importance of durables service flow, α takes a value of around 50%, except for the case controlling for the weather effect ($\alpha = 0.06$).

[Insert Table 7 here]

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5   Conclusion

In this paper, we develop a consumption-based capital asset pricing model that incorporates variation in the utilization of durable goods. Using CO_2 emissions, we identify the unobserved usage of durable goods and show that the risk associated with the time-varying utilization is important. While changes in durable goods stock are relatively smooth, variation in the utilization is more procyclical. This procyclicality explains the countercyclical variation in the equity premium. Our model delivers stronger cross-sectional pricing power than the CAPM and several CCAPMs. It alleviates the joint risk premium and implied risk-free rate puzzle and yields a relative risk aversion of 10.5 and a subjective discount rate of 0.98.

Several points should be reemphasized. First, restricted by the low frequency of CO_2 emissions data and the length of the period that energy (which leads to CO_2 emissions) becomes important in generating durable goods consumption, our sample contains only forty-one years. Tests using higher frequency data, upon availability, may therefore offer a better assessment of our model. Second, although our model explains the cross section of excess returns for the 25 Fama-French portfolios, as well as 30 other portfolios (sorted on earnings/price, cashflow/pric, and dividend yield), it does not perform well in pricing industry portfolios. Even though durable goods industries (e.g., durables, cars) have positive and significant loadings on the utilization factor, the overall factor risk premium is not significant. This seems to be puzzling as we would expect the utilization risk to help explain industry portfolios. Lastly, although we model household's durable goods utilization in a plain vanilla setting in this paper, extensions can be done, such as considering the habit formation.

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Figure 1: Household Consumption Patterns

Figure 1 presents changes in household consumption patterns. Panel A presents two time series: (1) durable goods stock relative to nondurable consumption (dashed line); and (2) energy-dependent durable goods stock relative to non-energy-dependent durable goods stock (solid line). Panel B presents the real net stock index of automobiles, light trucks, and appliances. The sample period is from 1930–2010.

Panel A: Household Consumption Components
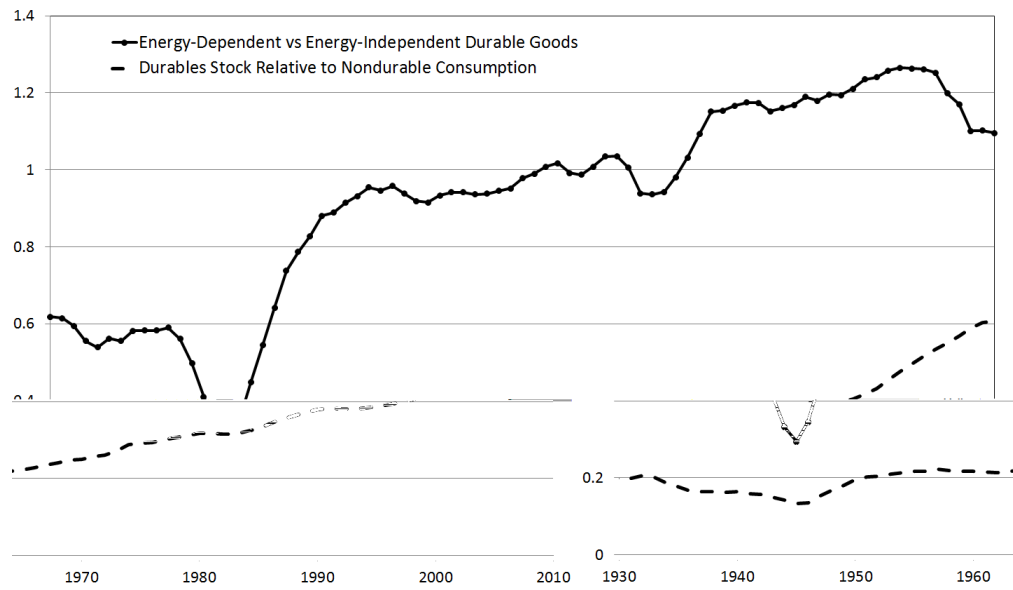
Panel B: Components of Energy-Dependent Durable Goods
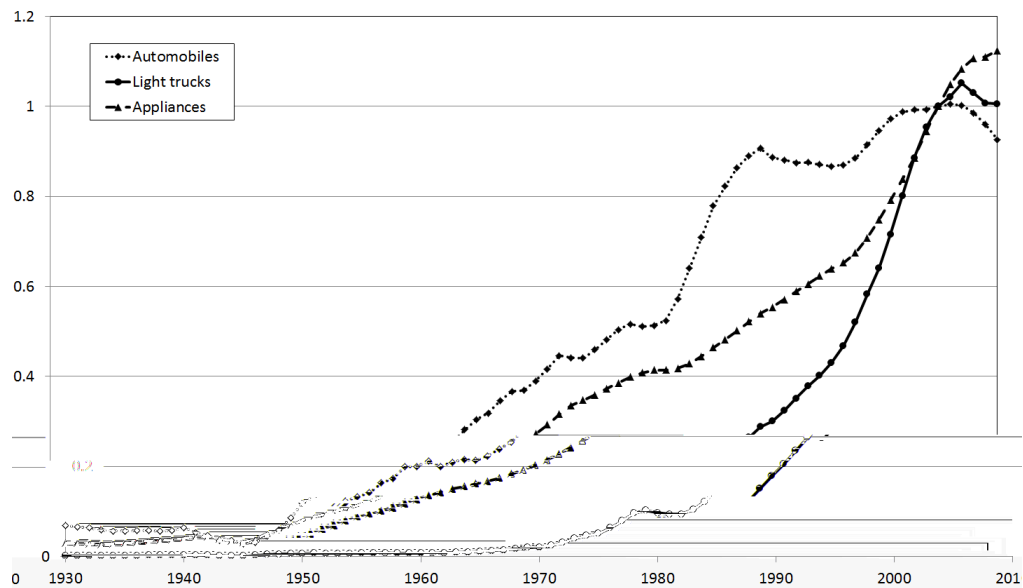
Figure 2: Time Series of Consumption Risks

Figure 2 shows the time series of the U-factor in comparison to the durables stock growth and the stock market real excess returns. The shaded bands indicate NBER recessions. The U-factor measures the time series variation in the utilization of durable goods. Growth rates are calculated using the "beginning-of-period" convention. Growth rates and returns are demeaned and scaled by standard deviation, and measured in percentage. The sample period is from 1970–2010.
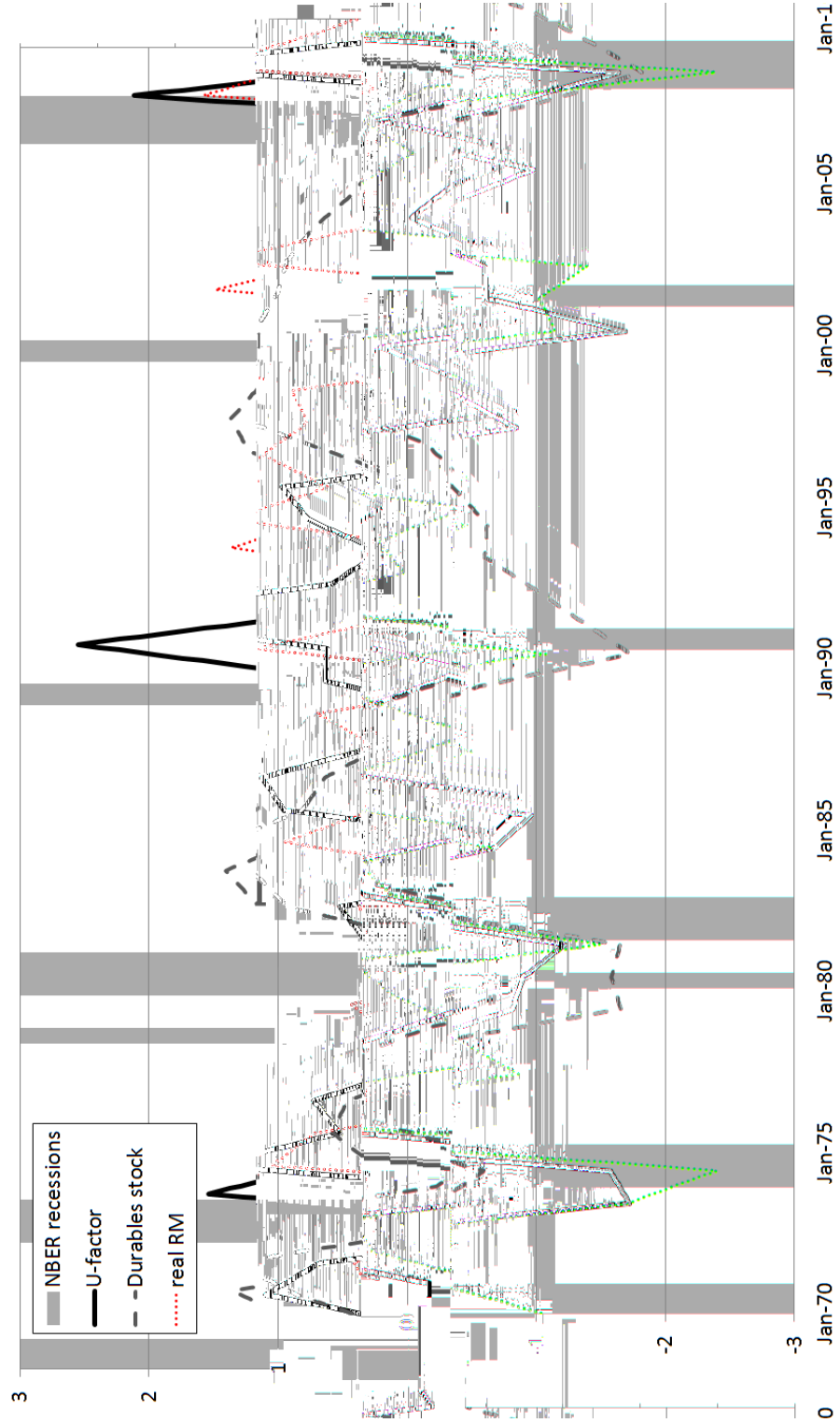
Table 1: Summary Statistics

This table presents summary statistics for various types of consumption risk: nondurable consumption growth, durables stock growth, and the U-factor. The U-factor captures the time series variation in the utilization of durable goods. Panel A provides variables' means, standard deviations, and AR(1) coefficients. Panel B reports pairwise correlations. PJ is the ultimate consumption growth with a three-year horizon proposed by Parker and Julliard (2005). Q4-Q4 is the fourth quarter consumption growth used in Jagannathan and Wang (2007). Growth rates are measured based on the beginning-of-period convention, except for Q4-Q4 given its definition. R^M is the market real excess return. The sample period is from 1970–2010, except for PJ (1970–2009).

Panel A: Descriptive statistics

	U-factor	Durables	Nondurables
Mean	0.0	3.9	1.7
SD	2.2	1.9	1.5
AR(1)	12.8	78.4	45.6

Panel B: Pairwise correlations

	U-factor	Durables	Nondurables
Durables	0.0		
Nondurables	40.5	63.4	
PJ	48.0	66.6	95.3
Q4-Q4	21.2	71.7	62.5
R^M	51.4	17.2	41.3

Table 2: GMM Estimation of Linear Factor Models

This table presents price of risk from GMM estimations under the four-factor model with U-factor, in comparison with the ones estimated under alternative models. Panel A presents the estimates of the four-factor model. Panel B presents estimates under a number of alternative models including the CAPM, the Fama-French three-factor model, a CCAPM with power utility, a CCAPM with Epstein-Zin preferences, Yogo's three-factor model, and a three-factor model with durables service flow. U-factor captures the time series variation in the utilization of durable goods stock. Newey-West three-lag adjusted t-statistics are in parentheses below the corresponding risk prices. Testing assets are the Fama-French 25 size and book-to-market portfolios. We report the implied preference parameters, including the relative risk aversion ( ), the elasticity of intertemporal substitution ( ), and the relative importance of durables service flow ( ). t-statistic of the preference parameters, in parentheses, are computed using the Delta method. Root-mean-squared errors (RMSEs) and p-value for the J-tests are reported in percentage, R^2 is defined as one minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns.

	Panel A: Benchmark model			Panel B: other models			
	4-factor Utilization model (a)	CAPM (b)	FF3F (c)	CCAPM (d)	CCAPM-EZ (e)	Yogo (f)	3-factor (SF) (g)
Nondurables (b_1)	24.22			89.08	140.70	89.55	55.25
	(2.70)			(10.30)	(39.31)	(6.15)	(15.57)
Durables (b_2)	41.63					30.88	45.56
	(9.06)					(3.77)	(7.23)
U-factor (b_3)	42.68						
	(5.38)						
Market (b_4)	-4.31	1.43	1.11		-3.09	-1.99	-5.16
	(-3.93)	(4.28)	(2.31)		(-6.09)	(-4.07)	(-6.81)
SMB			1.68				
			(1.29)				
HML			3.52				
			(7.66)				
	61.76			89.08	137.61	118.45	95.65
	(9.51)			(10.30)	(40.38)	(18.23)	(11.94)
	0.08				0.03	0.02	0.06
	(6.32)				(8.61)	(7.73)	(11.09)
	0.70					0.26	0.49
	(5.93)					(8.3)	(27.38)
RMSEs (%)	1.75	3.41	1.39	2.47	1.99	1.99	1.79
R^2	0.65	-0.35	0.78	0.29	0.55	0.54	0.63
p-value	96.82	99.09	97.84	99.05	98.49	97.85	97.95
Sample	1970–2010	1970–2010	1970–2010	1970–2010	1970–2010	1970–2010	1970–2010

Table 3: GMM Estimation of the Linear Four-Factor Model using alternative nondurable consumption growth measures

This table reports the GMM estimated price of risk and implied preference parameters of the linear four-factor model in which the nondurable consumption risk is measured using alternative measures. Three alternative measures of nondurable consumption risk are considered: Q4-Q4 is the fourth quarter consumption growth used in Jagannathan and Wang (2007); PJ is the ultimate consumption growth with a three-year horizon proposed by Parker and Julliard (2005); NDexUH is the per capita growth of nondurable goods and services consumption, excluding expenditure on utilities, fuel, and housing rents. t-statistics are calculated using Newey-West (1987) three-lag correction. We report the implied preference parameters, including the relative risk aversion ( ), the elasticity of intertemporal substitution ( ), and the relative importance of durables service flow ( ). t-statistic of the preference parameters, in parentheses, are computed using the Delta method. Root-mean-squared errors (RMSEs) are reported in percentage, and R^2 is defined as one minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns. The p-values for the Wald-test (test of equality between factor risk prices of durable goods and the U-factor) and for the J-test (test of overidentifying restrictions) are in parentheses.

| | Alternative nondurables measures | | |
	Q4-Q4	PJ	NDexUH
Nondurables (b_1)	26.88	15.09	66.36
	(3.61)	(1.47)	(6.57)
Durables (b_2)	55.14	54.67	25.48
	(8.37)	(7.85)	(2.47)
U-factor (b_3)	50.27	36.81	25.24
	(4.78)	(4.18)	(3.71)
Market (b_4)	-4.43	-4.76	-4.42
	(-3.97)	(-4.12)	(-4.63)
	77.59	65.00	87.42
	(9.39)	(11.82)	(16.17)
	0.07	0.08	0.06
	(7.21)	(5.59)	(6.58)
	0.73	0.87	0.30
	(9.51)	(5.60)	(2.63)
Wald-test ($b_2 = b_3$)	0.01	0.16	0.00
	(0.92)	(0.69)	(1.00)
RMSEs	1.65	1.54	1.53
R^2	0.68	0.75	0.73
J-test	10.71	10.18	10.60
	(0.97)	(0.98)	(0.97)
Sample	1970–2010	1970–2009	1970–2010

Table 4: Fama-MacBeth Regressions

This table reports results from the Fama-MacBeth two-pass regressions of the linear factor model. Panel A presents factor risk premia estimated under the four-factor model, using 55 portfolios over the sample periods of 1970–2010,

Table 5: GMM Estimation of the Euler Equations

This table reports preference parameters estimated from the Euler equations. Parameters include the elasticity of intertemporal substitution , the relative risk aversion , the subjective discount rate , the elasticity of substitution between durables service flow and nondurable consumption , and the weighting parameters $_1$ and $_2$. Panel A reports results using the unconditional moment conditions. Panel B reports results using the conditional moments with instruments. Instruments are second lags of nondurable consumption growth, durables stock growth, U-factor, dividend-price ratio, long-short yield spread, size spread, value spread, and a vector of ones. Testing assets are the Fama-French 25 size and book-to-market portfolios for the unconditional estimation, and Fama-French 3-factor for the conditional estimation. t-statistics with Newey-West (1987) three-lag correction are reported in parentheses.

	Panel A: Unconditional	Panel B: Conditional with instruments
	0.06	0.07
	(5.05)	(6.50)
	56.64	10.51
	(4.32)	(5.88)
	0.91	0.98
	(15.39)	(21.64)
	2.29	1.00
	(0.86)	(1.45)
$_1$	0.72	2.95
	(0.00)	(44.11)
$_2$	0.69	2.66
	(0.00)	(58.57)
Sample	1970–2010	1970–2010

Table 6: Fama-MacBeth Regressions: Robustness Tests

This table reports factor risk premia of the four-factor model estimated using the Fama-MacBeth two-pass regressions. Four risk factors include nondurable consumption growth, durables stock growth, market factor, and the U-factor. The U-factor is constructed after controlling for alternative sources of emissions. The four alternative sources of emissions are private fixed investment, GDP, the weather effect, and the time effect. Panel A presents results using 55 portfolios as testing assets, including the 25 portfolios formed on size and book-to-market ratio, 10 portfolios formed on cashflow/price, 10 portfolios formed on earnings/price, and 10 portfolios formed on dividend yield; Panel B presents results using the Fama-French 25 portfolios. A constant is included in the second-stage regression. Regression coefficients (factor risk premia) are reported, with t-statistics using the Newey-West (1987) three-lag correction in parentheses. Root-mean-squared errors (RMSEs) and adjusted R^2 are in percentage.

Panel A: Controlling for alternative source of emissions (55 portfolios)

Control variable	U-factor	Nondurables	Durables	Market	Constant	RMSEs	Adj. R^2
Investment	1.97	0.91	0.72	-5.06	7.62	1.41	54.63
	(2.01)	(1.28)	(1.05)	(-1.13)	(1.91)		
GDP	2.20	1.28	1.14	-0.11	2.71	1.36	54.98
	(2.03)	(1.87)	(1.80)	(-0.03)	(0.83)		
Weather	2.56	1.21	0.52	-1.95	4.69	1.37	56.21
	(1.82)	(1.63)	(0.70)	(-0.52)	(1.39)		
Time-trend	2.37	0.89	0.33	-6.34	9.28	1.29	55.36
	(2.80)	(1.20)	(0.47)	(-1.40)	(2.20)		

Panel B: 25 portfolios

Control variable	U-factor	Nondurables	Durables	Market	Constant	RMSEs	Adj. R^2
Investment	1.95	1.30	1.65	-5.23	7.19	1.11	54.91
	(3.37)	(1.62)	(2.17)	(-0.95)	(1.40)		
GDP	2.32	1.34	0.99	-3.34	5.63	1.14	54.79
	(3.76)	(1.71)	(1.32)	(-0.65)	(1.18)		
Weather	2.51	1.10	0.55	-7.03	9.91	1.11	55.00
	(3.80)	(1.35)	(0.70)	(-1.21)	(1.79)		
Time-trend	4.75	1.29	0.99	-6.04	8.87	1.18	54.61
	(2.88)	(1.63)	(1.33)	(-1.07)	(1.66)		

Table 7: GMM Estimation of the Linear Four-Factor Model: Robustness Tests

This table reports price of risk and implied preference parameters of the linear four-factor model using the GMM estimation. Four risk factors include nondurable consumption growth, durables stock growth, market factor, and the U-factor. The U-factor is constructed after controlling for alternative sources of emissions. The four alternative sources of emissions considered are private fixed investment, GDP, the weather effect, and the time effect. The Fama-French 25 size and book-to-market portfolios are the testing assets. t-statistics of the risk prices are calculated using Newey-West (1987) correction with three-lag. We report the implied preference parameters, including the relative risk aversion ( ), the elasticity of intertemporal substitution ( ), and the relative importance of durables service flow ( ). t-statistics of the preference parameters, in parentheses, are computed using the Delta method. Root-mean-squared errors (RMSEs) are reported in percentage, and R^2 is defined as one minus the ratio of the cross-sectional variance of the pricing errors to the cross-sectional variance of realized average portfolio returns.The p-values for the Wald-test (test of equality between factor risk prices of durable goods and the U-factor) and for the J-test (test of overidentifying restrictions) are in parentheses.

	Investment	GDP	Time-trend	Weather
Nondurables	41.40	54.27	37.39	39.81
	(3.72)	(5.48)	(3.59)	(2.04)
Durables	43.33	54.35	27.09	41.55
	(6.50)	(8.76)	(5.12)	(2.48)
U-factor	43.53	31.29	30.64	28.98
	(3.91)	(4.16)	(3.69)	(3.02)
Market	-6.19	-1.47	-4.43	-5.98
	(-4.09)	(-2.19)	(-3.96)	(-3.19)
	78.54	107.14	60.05	75.38
	(6.71)	(15.86)	(9.34)	(4.08)
	0.08	0.02	0.08	0.09
	(10.15)	(4.44)	(7.56)	(12.83)
	0.57	0.52	0.47	0.57
	(6.67)	(7.56)	(3.90)	(2.72)
Wald-test ($b_2 = b_3$)	0.00	0.26	0.00	0.03
	(1.00)	(0.61)	(0.95)	(0.86)
RMSEs	1.66	1.78	1.80	2.06
R^2	0.70	0.63	0.62	0.54
J-test	10.41	10.80	10.73	10.35
	(0.97)	(0.97)	(0.97)	(0.97)
Sample	1970–2009	1970–2010	1970–2010	1970–2010

Appendices

A   Derivation of the Euler Equations

A.1   CCAPM with Endogenous Durable Stock

In this version of the model, we keep everything else the same as the baseline model, but let the durable goods stock holding be endogenously chosen, and model the law of motion embedded in the stock of the durable good. Specifically, the evolution of the durable good stock follows Equation A.1, where D_{t-1} is the amount of the durable good stock from the previous period, $\eta \in (0, 1)$ is the depreciation rate, and E_t is the purchase of new durable good.

$$D_t = (1 - \eta)D_{t-1} + G_t \tag{A.1}$$

We set the price of the nondurable good as the numeraire and let P_t^D be the relative price of the durable good. The representative household faces a different intraperiod budget constraints because of the additional cost involved in purchasing a new durable good stock, $P_t^D E_t$.

$$W_t = C_t + P_t^D G_t + \sum_{i=0}^{N} z_{i;t}, \tag{A.2}$$

Given the household's current wealth W_t and its current stock of durable good D_{t-1}, the household makes consumption and portfolio choice $C_t, D_t, z_{i;t}$ to maximize its recursive utility subject to the budget constraints. Following Bansal, Tallarini, and Yaron (2004) and Yogo (2006), we can simplify the consumption and portfolio choice problem through a change of variable that treats the durable good stock as an additional "asset". By solving the household's recursive utility maximization problem, we arrive at the following Euler equations:

$$1 = E[M_{t+1}R_{i;t+1}] \quad (i = 1, \ldots, N)), \tag{A.3}$$

$$1 = E[\frac{u_{Dt}}{P_t u_{Ct}} + (1 - \eta)M_{t+1}\frac{P_{t+1}^D}{P_t^D}], \tag{A.4}$$

where M_{t+1} follow the same definition as in Equation 5, and u_{Ct} and u_{Dt} are the partial differential of the intraperiod utility function with respect to C_t and D_t.

A.2   CCAPM with Endogenous Utilization Rate

In comparison to the benchmark model, while keeping everything else the same, we let the choice of the durable good utilization rate (U_t) be part of the optimization problem. We set the price of the nondurable good as the numeraire and let P_t^U be the relative cost of converting the durable good stock into the service flow. As a result, the representative household faces a different intraperiod budget constraints, as in Equation A.5, to account for the additional cost $P_t^U S_t$.

$$W_t = C_t + P_t^U S_t + \sum_{i=0}^{N} z_{i;t}, \tag{A.5}$$

Given the household's current wealth W_t and its current stock of durable good D_{t-1}, the household makes consumption and portfolio choice $C_t, U_t, z_{i;t}$ to maximize its recursive utility subject to the budget constraints. When the stock of the durable good is exogenously given, the problem of choosing the utilization rate is equivalent to choosing the service flow from the durable good S_t. By solving the household's recursive utility maximization problem, we have the Euler equations:

$$1 \quad = \quad E[M_{t+1}R_{i;t+1}] \quad (i=1,\ldots,N)), \tag{A.6}$$

where M_{t+1} follow the same definition as in Equation 5. There is also another condition implied by the intraperiod first-order condition:

$$\frac{\partial \mathsf{u}_t/\partial S_t}{\partial \mathsf{u}_t/\partial C_t} = P_t^U = \frac{\alpha}{1-\alpha}(\frac{U_t D_t}{C_t})^{-1}. \tag{A.7}$$

B Details on Linear Approximation of the Euler Equation

B.1 Derivation of Equation 8

Let $m_{t+1} = log(M_{t+1})$,

$$\frac{M_{t+1}}{E[M_{t+1}]} = exp(m_{t+1} - log(E[M_{t+1}])), \tag{B.1}$$

$$= exp(m_{t+1} - log(E[exp(m_{t+1})])), \tag{B.2}$$

$$\approx exp(m_{t+1} - log(E[1 + m_{t+1}])), \tag{B.3}$$

$$= exp(m_{t+1} - log(1 + E[m_{t+1}])), \tag{B.4}$$

$$\approx exp(m_{t+1} - E[m_{t+1}]), \tag{B.5}$$

$$\approx 1 + m_{t+1} - E[m_{t+1}]. \tag{B.6}$$

The approximation from B.2 to B.3 and from B.5 to B.6 uses the rule of $exp(x) \approx 1 + x$. The approximation from B.4 to B.5 uses the rule of $log(1 + x) \approx x$.

Let $m_{t+1} = a - b'f_{t+1}$, where $f_{t+1} = (\Delta c_{t+1}, \Delta d_{t+1}, \Delta u_{t+1}, r_{W;t+1})'$, and $b = (b_1, b_2, b_3, b_4)'$ defined in Equation 9 to 12, we can get Equation 8.

C GMM Estimation Details

C.1 GMM for the Linear Factor Model

The pricing equation with R_t^e as the excess return is $E[M_t R_t^e] = 0$, then the moment conditions we use for the GMM estimation of linear factor model is given as follows (Cochrane (2005)):

$$E\begin{bmatrix} R_t^e - R_t^e b'(f_t - \mu_f) \\ f_t - \mu_f \end{bmatrix} = 0.$$

The parameters to estimate are (b', μ_f) with dimension $2k$, where k is the number of factors. The number of moment conditions is $N + k$, where N is the number of testing assets. We implement

two-step efficient GMM and follow Yogo (2006) to choose the first-step weighting matrix as:

$$A1 = \begin{bmatrix} kI_N & 0 \\ 0 & \hat{\Sigma}_{ff}^{-1} \end{bmatrix}.$$

We define $\Sigma_{ff} = E[(f_t - \mu_f)(f_t - \mu_f)']$, $\Sigma_{fi} = E[(f_t - \mu_f)R_{it}^e]$, and $\mu_f = E[f_t]$. We can calculate the estimation implied risk factor beta loadings $\beta = \Sigma_{ff}^{-1}\Sigma_{fi}$. Fix the elasticity of substitution between nondurable and durable consumption ϵ, and we can also calculate the implied preference parameters (γ, ψ, α) as a function of risk prices (b_1, b_2, b_3, b_4):

$$\gamma = b_1 + b_2 + b_4, \tag{C.1}$$

$$\psi = -\frac{b_4 - 1}{b_1 + b_2}, \tag{C.2}$$

$$\alpha = \frac{\epsilon b_2}{\epsilon b_1 + \epsilon b_2 + b_4 - 1}. \tag{C.3}$$

The t-statistics for implied preference parameters are calculated using the Delta method. We define the ith sample moment condition as $gT_i = \frac{1}{T}\Sigma_{t=1}^T(R_{i;t}^e - R_{i,}^e tb'(f_t - \mu_f))$ and asset $i's$ average excess return as $\bar{R}_i^e = \frac{1}{T}\Sigma_{t=1}^T R_{i;t}^e$. We have the root-mean-squared error as $RMSE = \sqrt{\frac{1}{N}\Sigma_{i=1}^N gT_i^2}$ and $R^2 = 1 - \frac{\frac{1}{N}\Sigma_{i=1}^N(gT_i - \frac{1}{N}\Sigma_{i=1}^N gT_i)^2}{\frac{1}{N}\Sigma_{i=1}^N(R_i^e - \frac{1}{N}\Sigma_{i=1}^N R_i^e)^2}$.

C.2   GMM Estimation with Euler Equations

The moment conditions for the GMM estimation with the Euler equations are given as:

$$E\begin{bmatrix} E_t[(M_{t+1}R_{0;t} - 1)I_t] \\ E[M_{t+1}R_{t+1}^e I_t] \end{bmatrix} = 0,$$

where I_t is the instrument variables for conditional estimations. There are $(N + 1)I$ moment conditions with N is the number of testing assets, and I is the number of instrument variables. Variables used in the estimation are indicated by $Z_t = C_t, S_t, R_{w;t}, R_{i;t}$; parameters to be estimated are indicated by $\Theta = \psi, \gamma, \epsilon, \beta, \alpha_1, \alpha_2$; and instrument variables used include lagged dividend/price ratio, lagged long-short yield spread, lagged log growth of nondurable consumption, lagged log

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growth of durable stock, and lagged log growth of U-factor.

D   Other Tables

Figure D.1: Time Series of Consumption Risk: 1930–2010

Figure D.1 shows the time series of the nondurable consumption growth, durables stock growth, stock market real excess returns, and the U-factor. The shaded bands indicate NBER recessions. The U-factor measures the time series variation in the utilization of durable goods. Growth rates are calculated using the "beginning-of-period" convention. Growth rates and returns are demeaned and scaled by standard deviations, and measured in percentage. The sample period is from 1930–2010.