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Informational feedback between voting and speculative trading ☆

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ABSTRACT

This paper develops a model to investigate the interaction between collective decision making in voting and financial speculation. Protesting voters demand policy reforms by voting against the incumbent, but too many opposing votes result in an unfavorable outcome: a political regime change. Traders speculate on the change of the political regime. The size of the speculation informs voters about the electorate's composition, thereby influencing the outcome of the election. We find that, in equilibrium, the strategic substitutability of protest voting makes speculations strategic substitutes via informational feedback, thereby incentivizing speculators to trade less on the correlated public signal. This strengthens the role of financial markets in providing information and amplifies the impact of the financial market's information on ex post political outcomes. We relate our theory to the Brexit referendum, and further discuss the robustness and limitations of our findings by considering more general information environments and voter preferences.

1. Introduction

Political elections influence public policy and, thus, can have a significant impact on financial markets. Financial markets, on the other hand, aggregate individual traders' dispersed information and may efficiently predict election outcomes.[1] In addition, ample evidence demonstrates that voters are paying close attention to financial market information.[2] This paper studies the informational linkage between the financial market and political decision making, as well as how that connection could determine the informational role played by the financial market and shape the political outcome.

To fix ideas, take the Brexit referendum as an example. The leave campaign won the majority vote, but it was a narrow victory (52% vs. 48%). Shortly after the referendum, the GBP/USD exchange rate fell more than 10% to a 31-year low, while

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[1] The literature on prediction markets (e.g., Wolfers and Zitzewitz, 2004) shows that markets provide the most efficient mechanism for predicting election outcomes.

[2] For example, based on a Morning Consult nationwide poll in the U.S., Rainey (2018) found that stock market volatility dominates voters' attention over political headlines.

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the GBP/EUR rate fell more than 7%.[3] Despite the dramatic market response, no significant currency shorting occurred prior to the Brexit vote, and exchange rates remained stable. Why was the financial market unable to forecast the outcome of the election? Is it possible that the financial market affected the referendum and contributed to Brexit's surprise victory?

Motivated by the example of Brexit, we develop a theoretical model to investigate how the informativeness and effectiveness of the financial market are shaped by the informational feedback between speculative trading and voting. In our model, traders speculate on a particular voting outcome — a political regime change — prior to an election by shorting the domestic currency, and voters extract information about the electorate's composition from the size of the speculation. This generates informational feedback between voting and speculative attacks. That is, on the one hand, speculators' informational choices will change the information aggregation in the financial market and, thus, determine voters' responses to that information; on the other hand, voters' responses will shape speculators' informational choices through their cumulative impact on a political regime change.

Our theoretical analysis demonstrates, perhaps surprisingly, that when speculators are more optimistic about a political regime change, the increased size of the speculation discourages voters from voting in favor of the change. As a result, speculative attacks are *strategic substitutes*, and speculators tend to differentiate their informational choices by trading less (more) on the correlated public signal (conditionally independent private signal). This facilitates information aggregation in the financial market. As such, voters become more responsive to financial market information, thereby magnifying the real impact of the financial market on voting; that is, financial market information becomes more influential, ex post, in terms of changing the political outcome.

More specifically, we consider *protest voting* according to the motivating example of Brexit.[4] voters are dissatisfied with current policies. However, they want the incumbent to stay in power so that necessary policy reforms can be implemented. These voters signal their dissatisfaction and demand necessary policy reform by casting their protest votes against the incumbent party if they are confident that the incumbent party will win. This “protest” will succeed, and the subsequent policy reform will occur only if a sufficient number of opposing votes are cast (say, more than 20%). However, too many opposing votes (say, more than 50%) will overthrow the political regime and result in political extremism (e.g., a party with an opposing ideology or extremist views will come into power). The expectation is that the political regime change will be followed by a radical policy shift, lowering voters' living standards and resulting in a significant devaluation of the domestic currency.

Protesting voters prefer policy reform to alternative voting outcomes — namely, incumbent wins in the absence of policy reform and a political regime change. Voters have idiosyncratic preferences that can be characterized by the extent to which they would suffer from a political regime change. If voters suffer less from this voting outcome, then we say their preference for political regime change is (relatively) (relat(116 1 Tf 9.9626 0 1 Tf 9.9626t3 15 9.926 0 0 9.9

learning channel is that a larger speculative attack would discourage protesting voters from voting against the incumbent regime.

This discouragement effme24464 6.9626 351.2738 686.2.23emen7 model changes protesting voters' estimated chances that their vote will be pivotal in two distinct pivotal states. In contrast, in the costly strategic voting model of Taylor and Yildirim (2010), information about other voters' preferences affects one's incentives to abstain (or free ride) and, consequently, affects voter turnout. In addition, the public information in our model is endogenously generated through financial market speculation, and it reveals only partial information about other voters' preferences.

Our paper also contributes to the literature on speculative currency attacks. This literature assumes direct payoff complementarity between speculators and highlights the self-fulfilling prophecy of speculative attacks (e.g., Obstfeld, 1996 and Morris and Song Shin, 1998).[7] A notable exception is Goldstein et al. (2011), which assumes away this direct payoff complementarity and considers the case in which a central bank learns from the speculative attack and endogenously decides to abandon a fixed exchange rate. Goldstein et al. (2011) establish the informational complementarity among speculators and show that, in equilibrium, speculators overweight the public signal. Similarly, our model does not include any direct payoff externalities between speculators; however, we examine a situation in which speculative currency attacks can provide useful information to a large number of voters. In contrast to previous research, we find that speculators' actions feature strategic substitutability in such an environment. According to Hellwig and Veldkamp (2009), under strategic substitutability, agents will differentiate that their informational choices. Consistent with their findings, our model

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reforms. However, if too many opposing votes are cast, then the political regime will change and will be replaced by political extremism (e.g., an ideological party will come into power).

The outcome of voting, denoted by $e(a_i, a_{-i})$, is determined by voter i's choice as well as by that of other voters, $a_{-i} \equiv (a_l)_{l \neq i}$. We denote the total number of votes as $M \equiv \#\{i | a_i = 1\}$, and, accordingly, the vote share is $m \equiv \frac{M}{N}$. There are three possible outcomes $e(a_i, a_{-i}) \in \{\mathcal{D}, \mathcal{R}, \mathcal{E}\}$ depending on m:

1. $\mathcal{D}$efault (status quo — no policy reform and no political regime change) if $m < p_l$;
2. $\mathcal{R}$eform (necessary policy reform but no political regime change) if $m \in [p_l, p_h)$;
3. $\mathcal{E}$xtremism (political regime change and radical policy shift) if $m \geq p_h$.

The parameters p_l and p_h, determined by the difficulty of calling for policy reform and voting rules, respectively, satisfy that $0 < p_l < p_h < 1$.

The voting outcome e yields U_i^e to voter i, and, accordingly, we can write voter i's payoff as

$$v_i(a_i, a_{-i}) = \sum_{e'=\mathcal{D},\mathcal{R},\mathcal{E}} \mathbb{1}\{e(a_i, a_{-i}) = e'\} U_i^{e'}. \tag{1}$$

We assume that policy reform (outcome $\mathcal{R}$) is the most favorable outcome to all voters; that is, $U_i^{\mathcal{R}} > \max\{U_i^{\mathcal{D}}, U_i^{\mathcal{E}}\}$ holds for all i.[10] To simplify our analysis, we introduce u_i to capture the preference of voter i,

$$u_i \equiv \ln\left(\frac{U_i^{\mathcal{R}} - U_i^{\mathcal{D}}}{U_i^{\mathcal{R}} - U_i^{\mathcal{E}}}\right). \tag{2}$$

Note that u_i increases with $U_i^{\mathcal{E}}$, and, in this sense, it measures the relative preference for a political regime change. Moreover, we assume that u_i is distributed following a commonly known normal distribution $\mathcal{N}(\theta, \sigma_u^2)$, in which θ captures the voterpre

Each speculator j at $t = 1$ chooses whether to short the currency ($d_j = 1$) or not ($d_j = 0$). We normalize the cost of speculation to $c \in (0, 1)$ and the return from a successful short position to 1. Therefore, for any

Next, we define a new mapping $\mu_k(\cdot)$ based on the speculator's choice k as follows[17]:

$$\mu_k(A) \equiv \frac{1}{1+k}\left(\frac{1}{\sqrt{\tau_s}}\Phi^{-1}(A) + s_0\right). \tag{4}$$

Based on expression (3), all relevant information about the fundamental θ revealed by the size of speculation A is summarized in the noisy public signal, $\mu = \mu_k(A)$; that is,

$$\mu = \mu_k(A(\theta, \varepsilon_p)) = \theta + \frac{k}{1+k}\sigma_p \varepsilon_p. \tag{5}$$

We denote the precision of this public signal as τ_μ. This precision

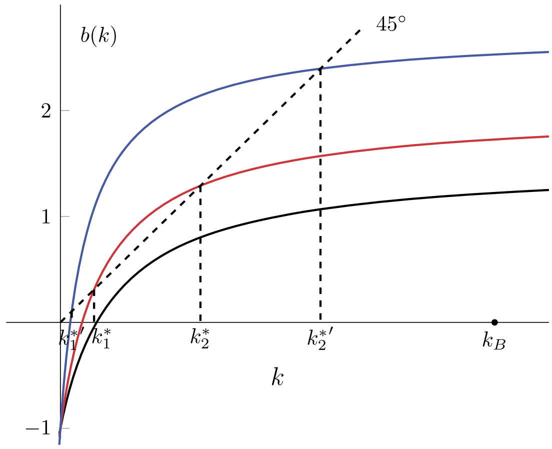
Proposition 1 gives the political regime change condition (11). From that condition, it should be clear that a higher realization of μ discourages voting and makes a political regime change less likely to occur. For speculators, shorting the domestic currency is profitable only if condition (11) holds. Therefore, this condition plays a vital role in solving the equilibrium speculation strategies.

3.2. Speculator's strategy in equilibrium

Let us now turn to solving for the speculator's strategy in equilibrium. When deciding how to trade on the private signal s_j and public signal s_p, speculators understand that the size of speculation be observed by voters and will affect the political regime change (see (11)). More formally, for any given strategy $_0$ chosen by other speculators, speculator that, from observing the size of speculation voters will learn

$$\mu \quad \mu\ (\ )\ \theta \quad \frac{}{1}\sigma\ \varepsilon \quad \frac{}{1}(\tag{12}$$

Compared with the private signal s_j, which



Note: This figure presents the best-response function $b(k)$ under the parameter values $\sigma_u = 3$, $\sigma_p = 3$, $\sigma_s = 6$, $z_h = 0$, $z_l = -1$ (blue curve), $z_l = -2$ (red curve), and $z_l = -3$ (black curve). The intersection of the blue (or red) curve with the 45 degree line presents the fixed points k^*. (For interpretation of the colors in the figure, the reader is referred to the web version of this article.)

Fig. 1. Function $b(k)$ and Fixed Points k^*.

$\delta(\tau_\mu)$, increases with the precision of this public signal. In a benchmark case in which no learning from the financial market takes place (i.e., $\delta(\tau_\mu = 0) = 0$), the regime change condition (11) is reduced to $\theta \geq \sigma_u z_h$. In the absence of informational feedback, there is no strategic interaction between speculators. Therefore, any speculator will choose a *Bayesian weight* $k_B = \frac{\tau_p}{\tau_s}$ on s_p, regardless of the other speculators' trading strategies. In another extreme case, in which learning is perfect (i.e., $\tau_\mu \to +\infty$), the magnitude of the discouragement effect is maximized at $\delta(\tau_\mu) \to 1$. In this limiting case, the best response is to trade against the public signal s_p to the maximum; that is, the assigned weight is -1.

Suppose that the precision of information τ_μ increases while other speculators' choices of k remain fixed. In this case, the discouragement effect is of a larger magnitude (i.e., $\delta(\tau_\mu)$ increases), and, therefore, μ is more negatively correlated with the political regime change (see (11)). As a result, the speculators who trade on the political regime change would place a lower weight on the public signal s_p, which is positively correlated with the realization of μ. That explains why $B(k, \tau_\mu)$ decreases with τ_μ.

Note that when speculators trade more on the public signal, the public signal μ becomes less informative about θ; that is, $\tau_\mu(k)$ decreases with k (see (6)). Following the logic discussed above, this would induce speculator j to put a higher weight on s_p; that is, $\frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu} \frac{\mathrm{d}\tau_\mu(k)}{\mathrm{d}k} > 0$. This demonstrates the first channel through which one's optimal choice of weight $b(k)$ is affected by the choices of others. This channel establishes the complementarity in speculators' choices of k, and it involves voters' learning efficiency and the magnitude of the discouragement effect.[23]

The second channel hinges on the substitutability in speculators' informational choices. Assume that τ_μ is fixed, as well as the magnitude of discouragement $\delta(\tau_\mu)$. Suppose that other speculators trade more on the public signal s_p (i.e., k increases). In this case, the size of speculation A and, accordingly, the public signal μ become more positively correlated with s_p (see (12)). Therefore, the likelihood of a regime change becomes more negatively correlated with s_p as a result of the discouragement effect. As a result, any rational speculator would assign a lower weight to s_p; that is, $\frac{\partial B(k,\tau_\mu)}{\partial k} < 0$. This demonstrates the *informational substitutability* among speculators. Because of informational substitutability, as long as other speculators assign a positive weight to s_p, as speculator j has an incentive to trade against this public signal, the optimal weight must be strictly lower than the Bayesian weight k_B. This explains the underlying intuition of property 2 in Lemma 4.

Interestingly, as the third property of Lemma 4 shows, the first channel that involves the magnitude of the discouragement effect always dominates the second one, which relies on this discouragement effect. Hence, the best response $b(k)$ always increases with k, and there is complementarity between speculators' choices of the weight placed on the public signal s_p. When k increases from 0, the complementarity becomes weaker as the discouragement effect weakens. That explains the concavity of $b(k)$.

Fig. 1 depicts the shape of the best-response function $b(k)$. We already know from Lemma 4 that $b(k)$ is an increasing and concave function for $k > 0$ with $b(0) = -1$ and $\lim_{k\to+\infty} b'(k) = 0$. Then, from Fig. 1, it should be clear that, depending

[23] Note that the precision of information provided by the financial market τ_μ in addition to affecting voters' learning, influences voters' decision rules in the strategic voting game through its impact on the magnitude of the discouragement effect $\delta(\tau_\mu)$. This can be seen from (9) and (11), in which the magnitude of the discouragement effect $\delta(\tau_\mu)$ determines voters' strategy and the regime change condition, respectively. In this sense, this channel highlights the difference between the case in which the regime change is determined by a single decision maker (for example, a central bank, as considered in Goldstein et al., 2011) and the case in which it is determined by the collective decision making of multiple agents.

3.4. Informativeness of the financial market

One critical component of informational feedback is that the voters learn from financial market speculation. The informativeness of the financial market, denoted as ρ_k, can be measured by the precision of the signal μ that summarizes all information voters learn from the financial market; that is,

$$\rho_{k^*} \equiv \tau_\mu(k^*) = (\frac{1+k^*}{k^*})^2 \tau_p. \tag{17}$$

Clearly, the financial market's informativeness is determined by how speculators trade on their information in equilibrium, or k^*.

The next proposition shows that informational feedback makes the financial market more informative, even though the information aggregation is not efficient.

Proposition 4 *(Financial Market's Informativeness). Compared with no learning, informational feedback makes the financial market more informative ex ante; that is,* $\rho_{k^*} \in (\rho_{k_B}, +\infty)$ *for* $k^* = k_1^*, k_2^*$.

Note that the information in the financial market, if efficiently aggregated, is able to perfectly reveal θ. To see this, regardless of the precision of both the public and private signals τ_p and τ_s if speculators trade only on private signal s_j (i.e., $k = 0$), then the size of the θ θ (τ

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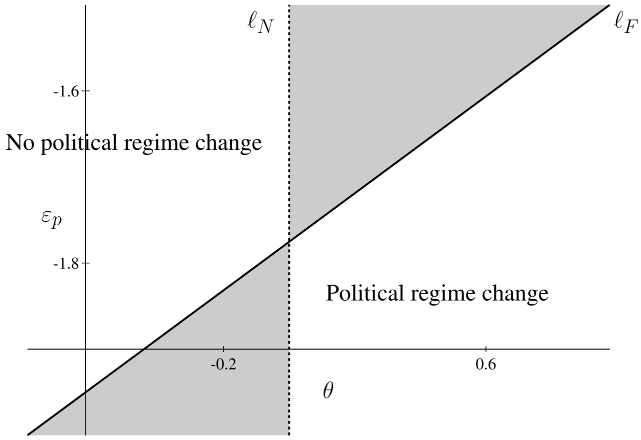

Note: The solid line ℓ_F presents the ex post regime change condition (18) based on the larger equilibrium weight k_2^*. The dashed line ℓ_N represents this condition without learning (i.e., $\theta \geq \sigma_u z_h$). The shaded areas represent the set of realizations (θ, ε_p) where the common noise ε_p changes the status of the political regime through informational feedback. The parameter values used in this figure are $\sigma_u = 3$, $\sigma_p = 3$, $\sigma_s = 6$, $z_h = 0$, and $z_l = -1$.

Fig. 2. The dependence of political regime change on θ and ε_p.

currency when the political regime changes, or they may mistakenly short the currency when no political regime change occurs. Indeed, this misguidance accounts for why speculators underweight the public signal s_p in equilibrium.

Recall that the informational feedback makes the financial market more informative (i.e., $\rho_{k^*} > \rho_{k_B}$). From condition (18), the ex post impact of ε_p on the political outcome has a magnitude of $\Lambda\sqrt{\rho_{k^*}}$. Therefore, the presence of informational feedback makes the common noise ε_p (or the financial market information) more influential in determining the political outcome.

Fig. 2 depicts how the change of political regime is determined by the realization of fundamental θ and the noise ε_p in the financial market, ex post. The solid line (with a positive slope) ℓ_F in Fig. 2 represents the regime change condition (18) associated with the larger equilibrium weight k_2^*.[26] The political regime changes ($e = \mathcal{E}$) if and only if the realization of (θ, ε_p) lies beo6 341.034 Tm (lies)Tj 4m 0 Tc ( )Tj /F1 1 Tf 7.9701 0 0 7.9701 250.6592 362.400j /4132u9701i626 170.4548

4. Discussion and extension

Thus far, we have constructed a model to study the informational feedback between financial market speculation and voting and have provided closed-form solutions to characterize the optimal strategy of voters and speculators. In this section, based on the solved equilibrium, we conduct comparative statics analysis and discuss the robustness and limitation of our theoretical results, taking into account more general setups regarding voter preferences and information structures.

4.1. Comparative statics

The tractability of the model enables clean comparative statics analyses. However, because of the multiplicity of equilibria, such analyses may not produce robust results. When parameters vary, it is possible that the economic agents in our model may switch from one equilibrium to another. In addition, different equilibria can have opposite comparative statics results. For example, as Fig. 1 shows, when $\Lambda = \sigma_u(z_h - z_l)$ varies, the equilibrium weights k_1^* and k_2^* change in opposite directions. Nonetheless, we would like to highlight one finding that, while inconclusive, can be useful in understanding how exogenous information affects financial market informativeness.

Suppose that the public signal s_p in the financial market becomes a more precise signal about the fundamental θ (i.e., τ_p increases). The conventional wisdom would be that such a changemarket

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The first important assumption we have made is that voters cannot access financial market information except for the size of speculation. Note that, if all signals possessed by the speculators are accessible to the voters, then the voters do not learn any additional information from the size of the speculation. Alternatively, if the voters can access the public signal s_p but not the private signals s_j in the financial market, then, in equilibrium, the size of speculation A would perfectly reveal the fundamental θ no matter how speculators trade on their information.[28] Therefore, if voters were able to observe the speculators' information directly, no informational feedback between speculative trading and voting would take place. In this sense, the inaccessibility of speculators' information to voters is an essential feature of our model.[29]

Another crucial feature of the information structure is that speculators possess some conditionally correlated signals about fundamental θ. In the benchmark model, for simplicity, we assume that all speculators share a public signal s_p, and, thus, the size of the speculation is dependent on the common noise ε_p. To see why this is crucial to our results, suppose that speculators have only conditionally independent signal(s). In this case, as we have discussed, the size of speculative trading A would effectively cancel out all conditionally independent noises and perfectly reveal the fundamental θ. However, a reasonable concern could be that since signal s_p is public, then it might also be observable to (some) voters. Below, we consider an alternative information structure to show that our findings do not rely on the particular information setting with the public signal s_p.

Consider an alternative information structure in which two private signals are available to each speculator j: (1) $s_j = \theta + \sigma_s\varepsilon_j$ (the same as our benchmark setting); and (2) $x_j = \theta + \sigma_q\varepsilon_q + \sigma_\eta\eta_j$, in which the private noises $\{\eta_j\}_j$ and the common noise ε_q are independently and identically distributed following standard normal distribution $\mathcal{N}(0, 1)$. We let $\tau_q \equiv \frac{1}{\sigma_q^2}$, $\tau_\eta \equiv \frac{1}{\sigma_\eta^2}$, and use $\tau_x \equiv \frac{\tau_q\tau_\eta}{\tau_q+\tau_\eta}$ to denote the precision of private signal x_i.[30] The following proposition presents the equilibria under this alternative information structure.

Proposition 7. *Under conditions $\sigma_u > z_h - z_l$ and $\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_x}\right) \geq 0$, there are two equilibria featured by $k_x^* = k_{x,1}^*, k_{x,2}^* \in (0, k_B' \equiv \frac{\tau_x}{\tau_s})$, in which $k_{x,1}^*$ and $k_{x,2}^*$ can be solved by replacing τ_p with τ_x in (16). In each equilibrium with k_x^*,*[31]

1. *the speculation strategy at $t = 1$ is $d_x^*(s_j, x_j) = \mathbb{1}\{s_j + k_x^* x_j \geq x_0(k_x^*)\}$,*
2. *the voting strategy at $t = 2$ is $a^*(u_i, \mu) = \mathbb{1}\{u_i \geq u^*(\mu)\}$ in which $u^*(\cdot)$ is given in (9), and*

$$\mu = \mu_{k_x^*}(A(\theta, \varepsilon_q)) \equiv \frac{1}{1+k_x^*}\left(\sqrt{\frac{\tau_s + k_x^{*2}\tau_\eta}{\tau_s\tau_\eta}}\Phi^{-1}(A(\theta, \varepsilon_q)) + x_0(k_x^*)\right) = \theta + \frac{k_x^*}{1+k_x^*}\sigma_q\varepsilon_q; \tag{19}$$

3. *and voters' posterior belief is given in (7) with $\mu = \mu_{k_x^*}(A)$ and $\tau_\mu = \left(\frac{k_x^*}{1+k_x^*}\right)^2 \tau_q$.*

It should be clear that, despite the absence of a public signal, this alternative information structure satisfies the two essential features we mentioned above. Since all private signals x_j contain the common noise ε_q, they are correlated across all speculators conditional on θ. As such, the common noise ε_q would affect each speculator's trading and thus the size of the speculation, whereas conditionally independent noises (i.e., ε_j and η_j) would be canceled out. For that reason, the informativeness of the financial market is dependent only on τ_q but not on the precision of the private signal x_i (i.e., τ_x). Other than this difference, as shown in Proposition 7, the equilibrium is qualitatively the same as in the baseline model if we replace the precision of the public signal s_p in the baseline model (i.e., τ_p) with the precision of the private signal x_j (i.e., τ_x). Therefore, we can draw the conclusion that all of our results carry over to this alternative information environment.

5. Conclusion

We develop a dynamic model in this paper to investigate the informational feedback between political decision making and financial speculation. The strategies of voting and speculation are jointly determined in our model. In equilibrium, the size of financial market speculation provides voters with information about the electorate's preferences. We demonstrate that strategic substitutability in protest voting leads to strategic substitutability in speculative attacks in equilibrium. As a result, speculators underweight the conditionally correlated signal but trade more on the conditionally independent signal. This strengthens the financial market's role in providing information; yet, it also increases the ex post impact of the financial market's noise, which can mislead speculators and contribute to an unfavorable electoral outcome. In such an environment,

[28] Note that, in any equilibrium, the voters understand how speculators trade on the private signal s_j and public signal s_p. If voters can also observe the realization of s_p, then they can perfectly infer the realization of θ from the size of speculation A.

[29] However, the fact that speculators have some information that is not accessible to voters does not necessarily mean that speculators are better informed than voters. As the voter's preference u_i is also informative about θ, it is ali T6(/F3 1 Tf 9.9626 0 0 9.9626 231.236111.8803 89920003)Tj /F1 1 Tf 6.3552 0 0 6.355

we find that more precise information does not always make the financial market more informative. We also discuss the essential features of the information structure upon which our results are based, as well as the robustness and the limitations of our results if we incorporate other types of voters who do not have protest demands.

Motivated by Brexit and for tractability concerns, we consider an economic environment in which the political outcome is determined by protest voting and voters extract information from the size of the speculation. We believe, however, that key insights gained from our study can be applied to other voting games (e.g., costly voting) or other learning technologies (e.g., learning from market prices). Nonetheless, there could be other forms of informational linkages between political decision making and financial markets beyond the one studied in this paper. For example, voting outcomes may provide traders with useful information about the future cash flows of certain financial securities, affecting the financial market's information aggregation; and financial market participants can trade as well as vote on corporate policies. These extensions, we believe, can be promising avenues for future research.

Declaration of competing interest

The authors hereby declare that they have nothing to disclose regarding funding sources, IRB approval or the any conflict of interests.

Appendix A. Omitted proofs

Proof of Lemma 1. The proof follows immediately that $u_i|\theta \sim \mathcal{N}(\theta, \frac{1}{\tau_u})$ and $\mu|\theta \sim \mathcal{N}(\theta, \frac{1}{\tau_\mu})$ based on the definition of $\mu = \mu_k(A)$ in (4) and the definition of $\tau_\mu = \tau_\mu(k)$ in (6), and the fact that the signals s_j and s_p are independent with u_i conditional on θ. □

Proof of Lemma 2. First, note that the expected payoff difference between attacking ($d_j = 1$) and not attacking ($d_j = 0$) is $\mathbb{P}(e = \mathcal{E}|s_j, s_p) - c$. To show strategic substitutability, it suffices to prove that $\mathbb{P}(e = \mathcal{E}|s_j, s_p; A)$ decreases with A.

Fix any choice of $\hat{u}(\cdot)$ and k, and consider any $A' > A''$. If $\hat{u}(\mu_k(A')) > \hat{u}(\mu_k(A''))$, then the vote share $m(\theta, \mu_k(A')) < m(\theta, \mu_k(A''))$ for any θ (see (10)). Therefore, for any s_j and s_p,

$$\mathbb{P}(e = \mathcal{E}|s_j, s_p; A') = \mathbb{P}(m(\theta, \mu_k(A')) \geq p_h|s_j, s_p) < \mathbb{P}(m(\theta, \mu_k(A'')) \geq p_h|s_j, s_p) = \mathbb{P}(e = \mathcal{E}|s_j, s_p; A'').$$

The same arguments can be used to prove strategic complementarity. □

Proof of Lemma 3. This proof is the same as Proposition 1 in Myatt (2016). Here, we reproduce the key steps for this result. Voter i chooses $a_i = 1$ if and only if

$$\begin{aligned} &\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{R}} + \mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{E}} \\ \geq &\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{D}} + \mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))U_i^{\mathcal{R}}. \end{aligned}$$

This is equivalent to

$$u_i = \ln\left(\frac{U_i^{\mathcal{R}} - U_i^{\mathcal{D}}}{U_i^{\mathcal{R}} - U_i^{\mathcal{E}}}\right) \geq \ln\left(\frac{\mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))}{\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))}\right). \tag{A.1}$$

The relative pivotal probability on the right-hand side of (A.1), for the case of $N \to +\infty$, can be solved as[32]

$$\ln\left(\frac{\mathbb{P}(\text{pivotal at } p_l|u_i, \mu, \hat{u}(\mu))}{\mathbb{P}(\text{pivotal at } p_h|u_i, \mu, \hat{u}(\mu))}\right) = \frac{z_l^2 - z_h^2}{2} + \ln\left(\frac{g(\hat{u}(\mu) + \sigma_u z_l|u_i, \mu)}{g(\hat{u}(\mu) + \sigma_u z_h|u_i, \mu)}\right),$$

Based on the payoff specification, speculator j would choose $d_j = 1$ if and only if the probability of a regime change is[33]

$$\mathbb{P}(e=\mathcal{E}|s_j,s_p) = \mathbb{P}\left(\theta \geq \frac{\delta(\tau_\mu)\left(\frac{k}{1+k}s_p - \frac{\sigma_u}{2}(z_h+z_l)\right)+\sigma_u z_h}{1-\delta(\tau_\mu)\frac{1}{1+k}}\middle| s_j, s_p\right) \geq c. \tag{A.4}$$

Note that, after observing s_j and s_p, speculator j forms a posterior belief: $\theta|s_j,s_p \sim \mathcal{N}(\frac{\tau_s s_j+\tau_p s_p}{\tau_s+\tau_p}, \frac{1}{\tau_s+\tau_p})$. Based on this belief, condition (A.4) is equivalent to

$$\sqrt{\tau_s+\tau_p}\left(\frac{\tau_s s_j+\tau_p s_p}{\tau_s+\tau_p} - \frac{\delta(\tau_\mu)\left(\frac{k}{1+k}s_p - \frac{\sigma_u}{2}(z_h+z_l)\right)+\sigma_u z_h}{1-\delta(\tau_\mu)\frac{1}{1+k}}\right) \geq \Phi^{-1}(c).$$

Based on the as definition of $B(k,\tau_\mu)$ and $s_0(k)$ (see (13) and (14), respectively), this condition can be written as $s_j + B(k,\tau_\mu)s_p \geq s_0(k)$.

Next, we prove the properties of the best response function $B(k,\tau_\mu)$. Recall that

$$B(k,\tau_\mu) = k_B - \frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(\tau_\mu)\frac{k}{1+k}}{1-\delta(\tau_\mu)\frac{1}{1+k}}.$$

Obviously, it is a differentiable function. Taking the derivative of B with respect to τ_μ, we have

$$\frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu} = -\frac{\tau_p+\tau_s}{\tau_s}\frac{k(1+k)}{(1+k-\delta)^2}\frac{\Lambda}{(1+\tau_\mu\Lambda)^2}, \tag{A.5}$$

which is strictly negative for any $k > 0$. Moreover, since $\delta(\tau_\mu = 0) = 0$, we have $B(k,\tau_\mu = 0) = \frac{\tau_p}{\tau_s} = k_B$. Similarly, as $\lim_{\tau_\mu\to+\infty}\delta(\tau_\mu) = 1$, we have $\lim_{\tau_\mu\to+\infty}B(k,\tau_\mu) = -1$, thereby proving the first property.

Taking the derivative of B with respect to other speculators' choice k, we have

$$\frac{\partial B(k,\tau_\mu)}{\partial k} = -\frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(1-\delta)}{(1+k-\delta)^2}. \tag{A.6}$$

As long as $\tau_\mu \neq 0$, we have $\delta(\tau_\mu) = \frac{\tau_\mu\Lambda}{1+\tau_\mu\Lambda} \in (0,1)$ and, therefore, the value of $\frac{\partial B(k,\tau_\mu)}{\partial k}$ is negative. Furthermore, it is easy to check that, for any $k > 0$, $\frac{\delta(\tau_\mu)\frac{k}{1+k}}{1-\delta(\tau_\mu)\frac{1}{1+k}} > 0$, and, thus, $B(k,\tau_\mu) < k_B$. This proves the second property.

Finally, we incorporate $\tau_\mu = \tau_\mu(k) = (\frac{1+k}{k})^2\tau_p$ into the function $B(k,\tau_\mu)$; that is,

$$b(k) = B(k,\tau_\mu(k)) = \frac{\tau_p}{\tau_s} - \frac{\tau_s+\tau_p}{\tau_s}\frac{(k+1)\tau_p\Lambda}{\left(1+\tau_p\Lambda\right)\left(k+\tau_p\Lambda\right)}. \tag{A.7}$$

Note that $b(k)$ is not continuous at the point of $k_0 \equiv -\frac{\tau_p\Lambda}{1+\tau_p\Lambda} \in (-1,0)$, whereby $\lim_{k\uparrow k_0}b(k) = +\infty$ and $\lim_{k\downarrow k_0}b(k) = -\infty$. However, we can show that in the range of $(-\infty,k_0)$ and $(k_0,+\infty)$, $b(k)$ is continuously increasing in k; that is,

$$\frac{\mathrm{d}b(k)}{\mathrm{d}k} = \frac{\partial B(k,\tau_\mu)}{\partial k} + \frac{\partial B(k,\tau_\mu)}{\partial \tau_\mu}\frac{\mathrm{d}\tau_\mu(k)}{\mathrm{d}k} = \frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(1-\delta)}{(1+k-\delta)^2} > 0.$$

Additionally, it is easy to check that $b(0) = -1$, $\lim_{k\to+\infty}\frac{\mathrm{d}b(k)}{\mathrm{d}k} = 0$, and that $b(k)$ is concave for $k > 0$ since, for $k \in (0,+\infty)$, $\frac{\mathrm{d}^2b(k)}{\mathrm{d}k^2} = -2\frac{\tau_p+\tau_s}{\tau_s}\frac{\delta(1-\delta)}{(1+k-\delta)^3} < 0$. □

Proof of Lemma 5. Any k^* that satisfies

$$k^* = b(k^*) = B(k^*,\tau_\mu(k^*)) = \frac{\tau_p}{\tau_s} - \frac{\tau_s+\tau_p}{\tau_s}\frac{(k^*+1)\tau_p\Lambda}{\left(1+\tau_p\Lambda\right)\left(k^*+\tau_p\Lambda\right)} \tag{A.8}$$

33 Note that this holds true only when $1-\delta(\tau_\mu)\frac{1}{1+k} > 0$. As such, this condition may not hold for some $k \in [-1,0]$. Consider $k \in [-1,0]$. If $1-\delta(\tau_\mu)\frac{1}{1+k} < 0$, then speculator j will choose $d_j = 1$ if and only if $s_j + b(k)s_p \leq s_0'(k)$, whereby the best-response function $b(k)$ remains the same. As will soon be shown, we cannot find any $k^* \in [-1,0]$ such that $b(k^*) = k^*$ so that any $k \in [-1,0]$ such that $1-\delta(\tau_\mu)\frac{1}{1+k} < 0$ cannot constitute an equilibrium. Another possibility is that $1-\delta(\tau_\mu)\frac{1}{1+k} = 0$. In this case, the regime change condition in (A.3) is independent of θ but is still dependent on s_p, and therefore the best response of speculator i is to trade only on the public signal s_p. This cannot support a symmetric equilibrium because that best response cannot be consistent with the other speculators' choice of $k \in [-1,0]$. For these reasons, taking into account the possibilities that $1-\delta(\tau_\mu)\frac{1}{1+k} \leq 0$ will not extend the set of equilibria.

must be a solution to the following quadratic equation:

$$(\Lambda + \frac{1}{\tau_p})k^{*2} + (2\Lambda - \frac{1}{\tau_s})k^* + \Lambda = 0. \tag{A.9}$$

It is obvious that we can find the solutions k_1^* and k_2^*, as shown in (16), under parameter condition (15). One way to prove $k_1^*, k_2^* \in (0, k_B)$ is to focus on the quadratic equation (A.9). Under condition (15), these two solutions exist, and they must satisfy that $k_1^* \cdot k_2^* = \frac{\Lambda}{\Lambda + \frac{1}{\tau_p}} > 0$, and

$$k_1^* + k_2^* = \frac{\frac{1}{\tau_s} - 2\Lambda}{\Lambda + \frac{1}{\tau_p}} \geq \frac{2\Lambda + 4\Lambda\frac{\tau_s}{\tau_p}}{\Lambda + \frac{1}{\tau_p}} > 0,$$

in which the first inequality comes from condition (15) and the second one is based on the fact that Λ, τ_s, and τ_p are all positive. Consequently, these two solutions must be strictly positive. By Property 2 in Lemma 4, we have $k^* = b(k^*) < k_B$ for $k^* = k_1^*, k_2^*$. □

In the following lemma, we present another perhaps more intuitive way of proving this result.

Lemma A.1. *Any solution to $k^* = b(k^*)$ must satisfy $k^* \in (0, k_B)$.*

Proof. Recall that $k_0 = -\frac{\tau_p \Lambda}{1+\tau_p \Lambda} \in (-1, 0)$, and $b(k)$ continuously increases with k in the range of $k \in (-\infty, k_0) \cup (k_0, +\infty)$. First, consider any $k \in (-\infty, k_0)$. From (A.7), we know that $\lim_{k \to -\infty} b(k) = (1 - k_0)\frac{\tau_p}{\tau_s} - k$

Next, let

$$f(\tau_p) \equiv 1 + \frac{1}{k_2^*(\tau_p)} - \frac{2}{\tau_p\left(\sqrt{\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_p}\right)}\right)} = \frac{\frac{1}{\tau_s} - \sqrt{\frac{1}{\tau_s^2} - 4\Lambda(\frac{1}{\tau_s} + \frac{1}{\tau_p})}}{2\Lambda} - \frac{2}{\tau_p\left(\sqrt{\frac{1}{\tau_s^2} - 4\Lambda\left(\frac{1}{\tau_s} + \frac{1}{\tau_p}\right)}\right)}.$$

Given $k_2^* > 0$, $\mathrm{sgn}(\frac{\mathrm{d}\rho_{k_2^*}}{\mathrm{d}\tau_p}) = \mathrm{sgn}((1 + \frac{1}{k_2^*}) f(\tau_p)) = \mathrm{sgn}(f(\tau_p))$. By definition of $f(\cdot)$, we have $f(\underline{\tau_p}) = -\infty$ and $\lim_{\tau_p \to +\infty} f(\tau_p) = \frac{\frac{1}{\tau_s} - \sqrt{\frac{1}{\tau_s^2} - 4\Lambda\frac{1}{\tau_s}}}{2\Lambda} > 0$. Furthermore, $f(\cdot)$ is a strictly increasing function because

$$\frac{\mathrm{d}f(\tau_p)}{\mathrm{d}\tau_p} = \frac{1}{\tau_p^2\sqrt{\frac{1}{\tau_s^2} - 4\Lambda(\frac{1}{\tau_s} + \frac{1}{\tau_p})}} + \frac{4\Lambda}{\tau_p^3\left(\frac{1}{\tau_s^2} - 4\Lambda(\frac{1}{\tau_s} + \frac{1}{\tau_p})\right)^{\frac{3}{2}}} > 0.$$

Therefore, there exists a unique $\tilde{\tau}_p \in (\underline{\tau_p}, +\infty)$ such that $f(\tilde{\tau}_p) = 0$. Based on monotonicity, for any $\tau_p < \tilde{\tau}_p$, $f(\tau_p) < 0$ and, accordingly, $\frac{\mathrm{d}\rho_{k_2^*}}{\mathrm{d}\tau_p} < 0$. This completes the proof. □

Proof of Proposition 7. First, under any strategy $d_j = \mathbb{1}\{s_j + k_x x_j \geq x_0(k)\}$, the size of the speculation is

$$A(\theta, \varepsilon_q) = \Phi\left(\sqrt{\frac{\tau_s\tau_\eta}{\tau_s + k_x^2\tau_\eta}}\left((1 + k_x)\theta + k_x\sigma_q\varepsilon_q - x_0(k)\right)\right).$$

As such, we can construct the public signal μ (see (19)):

$$\mu = \mu_{k_x}(A) = \theta + \frac{k_x}{1 + k_x}\sigma_q\varepsilon_q = \frac{1}{1 + k_x}\theta + \frac{k_x}{1 + k_x}(x_j - \sigma_\eta\eta_j), \tag{A.10}$$

and its precision is $\tau_\mu = (\frac{k_x}{1+k_x})^2\tau_q$. Based on the new definition of μ and τ_μ, the posterior belief of the voters is the same as $G(\theta|u_i, \mu)$ given in (7) and, thus, the voters' equilibrium strategy features the same $\hat{u}(\cdot)$ as the one present in Lemma 3. Accordingly, the condition for $e = \mathcal{E}$ is identical to the one given in (11).

Any speculator j, after observing s_j and x_j, forms a posterior belief of θ and $\sigma_\eta\eta_j$ as follows[34]:

$$\theta|s_j, x_j \sim \mathcal{N}(\frac{\tau_s s_j + \tau_x x_j}{\tau_s + \tau_x}, \frac{1}{\tau_s + \tau_x}), \text{ and } \sigma_\eta\eta_j|s_j, x_j \sim \mathcal{N}(\frac{\tau_q\tau_s(x_j - s_j)}{\tau_q\tau_s + \tau_q\tau_\eta + \tau_s\tau_\eta}, \frac{1}{\tau_\eta + \frac{\tau_s\tau_q}{\tau_s+\tau_q}}). \tag{A.11}$$

Plugging μ into the regime change condition, $e = \mathcal{E}$ occurs if and only if

$$\theta + \frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1 - \delta(\tau_\mu)\frac{1}{1+k_x}}\sigma_\eta\eta_j \geq \frac{\delta(\tau_\mu)\left(\frac{k_x}{1+k_x}x_j - \frac{\sigma_u}{2}(z_h + z_l)\right) + \sigma_u z_h}{(1 - \delta(\tau_\mu)\frac{1}{1+k_x})}.$$

Based on the posterior belief, speculator j would choose $d_j = 1$ if and only if

$$\mathbb{P}\left(\frac{\tau_s s_j + \tau_x x_j}{\tau_s + \tau_x} + \frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1 - \delta(\tau_\mu)\frac{1}{1+k_x}}\frac{\tau_q\tau_s(x_j - s_j)}{\tau_q\tau_\eta + \tau_s\tau_\eta + \tau_q\tau_s} \geq \frac{1}{\Omega}\frac{\delta(\tau_\mu)\left(\frac{k_x}{1+k_x}x_j - \frac{\sigma_u}{2}(z_h + z_l)\right) + \sigma_u z_h}{1 - \delta(\tau_\mu)\frac{1}{1+k_x}}\middle| s_j, x_j\right) \geq c,$$

in which Ω presents the standard deviation of the posterior distribution of $\theta + \frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1-\delta(\tau_\mu)\frac{1}{1+k_x}}\sigma_\eta\eta_j$ conditional on s_j and x_j, which can be solved following (A.11). Abusing the notation of $b(\cdot)$ and $B(\cdot, \cdot)$, the above condition can be rewritten as $s_j + b(k_x)x_j \geq x_0(k_x)$, in which

$$b(k_x) = B(k_x, \tau_\mu) \equiv \frac{\tau_x}{\tau_s} - \frac{\tau_q + \tau_s + \frac{\tau_x}{\tau_\eta}}{\frac{1+k_x}{k_x\tau_\mu\Lambda}(1 + \frac{\tau_q}{\tau_\eta})\tau_s + \tau_s}, \tag{A.12}$$

and

[34] For the posterior belief of $\sigma_\eta\eta_j$, one can think in the following way: speculator j has a prior belief $\sigma_\eta\eta_j \sim \mathcal{N}(0, \sigma_\eta^2)$ and also a noisy signal $(x_j - s_j)$, whereby $\sigma_\eta\eta_j = (x_j - s_j) - \sigma_q\varepsilon_q + \sigma_s\varepsilon_j \sim \mathcal{N}(x_j - s_j, \sigma_q^2 + \sigma_s^2)$.

$$x_0(k_x) \equiv \sqrt{\frac{1}{\tau_s+\tau_x} + \left(\frac{\delta(\tau_\mu)\frac{k_x}{1+k_x}}{1-\delta(\tau_\mu)\frac{1}{1+k_x}}\right)^2 \frac{1}{\tau_\eta + \frac{1}{\sigma_q^2+\sigma_s^2}}}\,\frac{\tau_x+\tau_s}{\tau_s}\Phi^{-1}(c) + \frac{\tau_s+\tau_x}{\tau_s}\frac{\sigma_u z_h - \frac{\sigma_u}{2}\delta(\tau_\mu(k_x))(z_h+z_l)}{1-\delta(\tau_\mu(k_x))\frac{1}{1+k_x}}. \quad \text{(A.13)}$$

Following the same procedures as we did for Lemma 4 and letting $k'_B \equiv \frac{\tau_x}{\tau_s}$, one can easily check that all properties hold true for the new function $B(k_x, \tau_\mu)$. Plugging $\frac{1}{\tau_\mu} = (\frac{k_x}{1+k_x})^2 \frac{1}{\tau_q}$ into (A.12), the solution k_x^* must satisfy that

$$k_x^* = b(k_x^*) = \frac{\frac{k_x^*}{\Lambda(1+k_x^*)} - \tau_s}{\frac{k_x^*}{\tau_q \Lambda(1+k_x^*)}(\tau_q+\tau_\eta)+\tau_\eta}\frac{\tau_\eta}{\tau_s}.$$

This condition can be simplified to

$$(\Lambda + \frac{1}{\tau_q} + \frac{1}{\tau_\eta})k_x^{*2} + (2\Lambda - \frac{1}{\tau_s})k_x^* + \Lambda = 0,$$

which is the same as (A.9) if we replace $\frac{1}{\tau_p}$ in (A.9) with $\frac{1}{\tau_q} + \frac{1}{\tau_\eta} = \frac{1}{\tau_x}$. Therefore, by replacing τ_p with τ_s in (15) and (16), we have the condition for the existence of k_x^* and the explicit solution of k_x^*, respectively. Following the same arguments as in the proof of Lemma 5, we can show that $k_x^* \in (0, k'_B)$. □

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