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Impediments to Financial Trade: Theory and Applications

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We propose a tractable model of an informationally inefficient market featuring nonrevealing prices, general preferences and payoff distributions, but not noise traders. We show the equivalence between our model and a substantially simpler one in which investors face distortionary investment taxes depending on both their identity and the asset class. This equivalence allows us to account for such phenomena as underdiversification. We further employ the model to assess approaches to performance evaluation and find that it provides a theoretical basis for some intuitive practices, such as style analysis, that have been adopted by finance professionals. (*JEL* G11, G12, G14)

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We present a simple, tractable framework featuring informational asymmetries in a multiasset economy. By incorporating strategic behavior for a subset of investors, our framework can dispense with the common assumption of "noise" traders (or random endowments), and does not rely on constant absolute risk aversion (CARA) preferences or normality of payoffs.[1] Moreover, the

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model is particularly tractable: it is isomorphic to a symmetric information one featuring investor- and asset-class-specific distortionary and redistributive taxes, reflecting investors' abilities to distinguish between good- and bad-

Our setup does not require noise traders to avoid information revelation through prices, due to two main assumptions. First, the swindlers are not competitive: each of them is endowed with a large holding of a fraudulent firm, and takes into account the effect of her trade on its price. Second, there is no short selling in equilibrium.[2] Consequently, a swindler can trade to push the price of her firm toward the pooling price, and the most a well-informed agent can do is withhold demand for this firm.

Investors inside the model have an incentive to bias their portfolios toward the locations where they enjoy an informational advantage, because those are the locations where they perceive lower implicit taxes. In contrast to the literature, which we summarize below, the portfolio biases toward specific locations exist independently of the particular realization of the signals. Furthermore, the portfolio of any given investor is "sparse," in the sense that it involves zero holdings in several individual assets, and may even involve zero allocations to entire asset classes, consistent with some features of real-world portfolios.[3]

The combination of nonrevealing prices (leaving room for the better informed investors to earn superior returns) and portfolio heterogeneity makes this model a natural framework to study the validity of different performance evaluation approaches from the perspective of an uninformed econometrician. An established literature has addressed this issue in noisy rational expectations models. Our framework, however, provides a novel way to capture situations in which superior performance is purely associated with selectivity rather than market timing, since assets at a location look identical to an outside econometrician, and no agent has superior information about the return distribution of the asset class itself.

We arrive at the following conclusions. Jensen's alpha may fail to identify informational advantage: passive strategies (i.e., returns obtained by simply buying the portfolio of all firms in a location, ignoring any signals) generically may have alpha, and informed strategies may have negative alpha. We link these phenomena to the heterogeneity of informational inefficiency across markets.

We then address the question of how to appropriately perform performance evaluation in our setup. We show that the key feature of successful performance evaluation is to use a criterion that assigns zero alphas to linear combinations of passive investments in the asset classes in which the informed investor participates actively. Intuitively, this ensures that the return obtained by an informed investor could not have been replicated by a passive investor investing in the same asset classes.

This is the reason the "style-alpha" approach, which was proposed by Sharpe (1992) and has proved very popular among practitioners, has several theoretical

[2] In the body of the text, we impose a shorting restriction. In the appendix we allow short selling and the swindler to manipulate the earnings of her company. We show that the ability to manipulate earnings deters shorting, even when it is allowed in principle.

[3] See Koijen and Yogo (2016) for empirical evidence on portfolios held by institutional investors.

merits in our framework. Such an approach identifies skill with the alpha obtained from a regression of the investor's return on the passive returns obtained in the asset classes where the investor participates actively. We show that the alpha of such a regression provides a clear mapping to the investor's informational advantage.

We also discuss the implications of market segmentation (and, more broadly, portfolio specialization) for performance evaluation. We argue that, for portfolios invested in a limited set of asset classes, the performance evaluation criterion should only be required to assign zero alphas to passive returns in this set, rather than in all asset classes. We illustrate this point with an example of a nonexploitable arbitrage, whereby it is impossible to use one pricing kernel to price all passive strategy returns, but it is still possible to evaluate performance using investor-specific evaluation criteria.

The paper relates to various strands of the literature. First, there is the literature incorporating noisy rational expectations equilibria (REE), which is the most popular approach to introduce informational asymmetries into finance models. This literature is too voluminous to summarize, so we provide indicative examples only. Technically, our setup borrows elements from both Grossman and Stiglitz (1980) and Akerlof (1970). Admati (1985) extends the noisy REE framework to a multiasset framework. This literature typically utilizes random supply shocks ("noise") to preclude revelation.[4] Moreover, our tax equivalence result does not require a CARA-normal framework.[5]

A popular application of multiasset REE models is the explanation of portfolio biases.[6] The issue of portfolio biases (in particular, the home bias) is especially prevalent in international finance, but the insights of this literature apply to understanding portfolio concentration and underdiversification more broadly. The common thread of that literature is that locals receive a superior signal about the aggregate performance of the local stock market.[7] The superior signal quality makes domestic agents face lower variance when investing in local stocks, leading to an *unconditional* home bias. A counterfactual implication is that conditional on a bad signal about the domestic market, locals should shun, if not outright short, domestic stocks. This seems at odds with the fact that the home bias is present for any given year, any given country, and for any sample period that one may consider. In our model the portfolio bias toward asset classes where one is better informed applies independently of any

4 However, (privately known) endowment shocks can achieve an outcome similar to that of random supply. See, for example, Diamond and Verrecchia (1981).

5 See also Breon-Drish (2015) for an analysis of REE frameworks without normal distributions.

6 Indicative examples are Gehrig (1993) and Brennan and Cao (1997). Relatedly, Van Nieuwerburgh and Veldkamp (2010) propose an approach relying on bounded information-processing capacity.

7 This is either outright assumed or the result of endogenous information acquisition. For instance, in Van Nieuwerburgh and Veldkamp (2009) local investors only need to have an arbitrarily small initial informational advantage in their local assets to generate a home bias, because they will endogenously choose to allocate their information acquisition capacity to the local asset.

specific realization of the signals. The reason is that the portfolio bias is not driven by having superior information about the aggregate dividend realization in a given location, but rather because of superior asset selection ability within the location.[8] This superior selection ability acts as a redistributive tax with obvious deterrence effects on investors who are not as well informed as locals.[9]

The sparsity of individual portfolios in our framework is another important qualitative difference from the REE literature, where all holdings are interior. While in principle one could obtain sparse portfolios in a conventional REE framework by introducing shorting constraints, this would jeopardize the tractability of the conventional REE framework, especially in a multiasset framework.

The international finance literature has modeled financial frictions as actual taxes or transaction costs, and on occasion informational disadvantages as taxes in reduced form.[10] Our paper provides the theoretical underpinning of doing so and draws attention to the proper specification of market-clearing conditions to ensure the correct mapping between redistributive taxes and asymmetric information frictions.

The paper also contributes to the literature that critiques CAPM alpha, estimated from the perspective of an uninformed investor, as a measure of skill—(see, e.g., Admati and Ross 1985; Dybvig and Ross 1985; Grinblatt and Titman 1989; Mayers and Rice 1979; among many others). Our results in Section 2.3 differ from those in Mayers and Rice (1979) and Dybvig and Ross (1985), who introduce the notions of timing and selectivity skill and show that when agents possess "pure selectivity," thus no "timing," skill (as they do in our framework), the alpha of an informed investor's portfolio return with respect to any reference portfolio must be nonnegative. Unlike in these papers, in our setting optimal informed portfolios are not necessarily interior (because of the shorting constraint), and we show that, as a result, they may have negative alpha even when an investor possesses pure selectivity skill. More generally, the ease with which our framework accommodates portfolio constraints allows us to show that the results in Mayers and Rice (1979) and Dybvig and Ross (1985) hinge critically on the (implicitly assumed) absence of such constraints. Besides studying the alpha of an investor's entire portfolio, we also discuss the alpha obtained by an investor in a single asset class (or, more generally, a given subset of asset classes). We are motivated by the real-world fact that portfolio

[8] Hatchondo (2008) also considers an adverse-selection setup. An important difference from that model is that we do not rely on noise trading and instead assume the existence of strategic "swindlers." In addition, our model accepts closed-form solutions and leads to a tax equivalence result. Finally, we can obtain the no-shorting outcome endogenously, although in the main body of the paper we directly impose short-selling restrictions for simplicity.

[9] See also Kurlat (2013) for the role of information asymmetry as taxation in a different example. Li et al. (2012) also model fraudulent assets, but in a different context.

[10] See, for example, Okawa and van Wincoop (2012) and the references therein for an illustrative example.

choice is routinely delegated to managers with mandates to pick good assets within a narrow set of asset classes.[11]

Sharpe (1992) proposed style analysis as a performance evaluation criterion. In some ways our paper provides an explicit micro foundation for this criterion in an equilibrium framework. We note, though, that the specific equilibrium return properties obtaining in our model are not identical to the ones assumed by the statistical model of Sharpe (1992).

We also relate to a literature that analyzes general properties of evaluation criteria and the use of stochastic discount factors for performance evaluation.[12] We differ in focus from that literature: Rather than considering any possible information structure, we make specific assumptions, which in particular allow a conceptual separation between diversifiable asset-selection risk (with agents being asymmetrically informed about it) and nondiversifiable asset class risk (with agents being symmetrically informed about it). Our framework results in a tighter theoretical characterization of valid performance measures—indeed, in an essentially unique performance evaluation criterion. With our assumptions, an essentially sufficient condition for a valid performance evaluation criterion is to assign a zero value to any linear combination of passive strategy returns in the asset classes where the investor is actively participating.

We conclude with two caveats about our conclusions on performance evaluation. First, we abstract from timing ability, that is, superior information about the behavior of asset classes as a whole, on which there exists an extensive literature. Instead, we concentrate on the stronger results that obtain when information pertains exclusively to the relative quality of individual securities. Second, our focus is exclusively on identifying a market participant's stock selection ability from the perspective of an econometrician. We therefore do not address how an individual investor would allocate her investment among various (potentially informed) managers, which is the central issue in the study of fund flows.[13]

1. Model

1.1 Locations, preferences, and firm and investor types

Time is discrete. In the baseline version of the model there are two dates, but we also consider a multiperiod version in Section 1.7.2 and Online Appendix A. All trading takes place at time $t=0$, while at $t=1$ all payments are made and consumption takes place. There are K different locations, and each investor is

[11] A voluminous literature studies how delegated portfolio choice may lead to portfolio weighting distortions. We do not attempt to summarize this literature and simply refer readers to Bhattacharya and Pfleiderer (1985) for an early and important contribution.

[12] See, for example, Chen and Knez (1996) and Glosten and Jagannathan (1994) for two indicative examples.

[13] For a discussion of these issues, see Berk and Green (2004), Ferson and Lin (2014), and Berk and van Binsbergen (2015), among others.

located in one of the K locations. There is a continuum of investors in each location and we index a representative investor in a given location by i. Investors maximize expected utility of period-1 wealth, $\mathrm{E}[U(W)]$, for some increasing and concave function U.

Investors' time-zero endowments consist of shares in firms that are domiciled in their location. Investors in every location i are of two types, common investors and swindlers, while firms are of two types, regular and fraudulent. The number of shares in each firm is normalized to one, as are the measures of investors and firms at each location.

Common investors in location i are a fraction $\kappa \in (0,1)$ of the population in that location. They are identically endowed with an equal-weighted portfolio of all regular firms in location i. All regular firms in location i produce the same random output D_i, and pay it out as a dividend. (Adding a firm-specific, idiosyncratic risk would be simple, but would offer no additional insights). The total measure of regular firms is κ in each location.

Swindlers are a fraction $1-\kappa$ of the population in each location. Each swindler is endowed with the share of one fraudulent firm. Fraudulent firms produce no output or dividend ($D_i = 0$).

For every firm in every location, there is a market for shares where any investor can submit a demand. Moreover, there exists a market for a riskless bond, available in zero net supply. The interest rate is denoted by r.

1.2 Signals

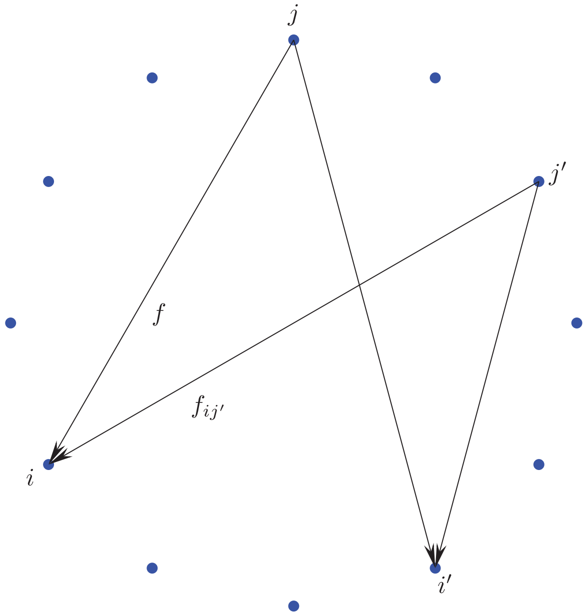
Each investor obtains a binary signal of the type—regular or fraudulent—of every firm in every location. The precision of these signals depends on the locations of the investor and the firm.

Specifically, an investor l in location i obtains a signal $\iota_{jk}^{il} \in \{0,1\}$ about every firm k in location j. This signal characterizes the firm as either regular ($\iota_{jk}^{il} = 1$) or fraudulent ($\iota_{jk}^{il} = 0$). An imperfect signal correctly identifies every regular firm as such. However, the signal fails to identify all fraudulent firms: it correctly identifies a fraudulent firm with probability π_{ij} and misclassifies it as regular with probability $1-\pi_{ij}$. For simplicity, all signals, that is, across all agents and all firms in all locations, are independent conditional on the firm types. Similarly, we assume $\pi_{ii} = 1$, so that investors are fully informed about their local markets. These assumptions are inessential and can be easily relaxed.[14]

Given this setup, Bayes' rule implies that the probability that a firm in location j is fraudulent given that investor i's signal identifies it as regular is given by

$$f_{ij} \equiv \frac{(1-\pi_{ij})(1-\kappa)}{\kappa + (1-\pi_{ij})(1-\kappa)}. \tag{1}$$

[14] If we wanted to remove the property $\pi_{ii} = 1$, we also would have to assume that different subsets of the continuum of local regular investors are endowed with different subsets of the continuum of regular firms.


j
j'
f
$f_{ij'}$
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1.4 Optimization problem

Common investors are price-takers. Taking as given a set of prices for risky assets for all firms in all locations and an interest rate, a common investor maximizes

$$\max_{B^{ci}, X^{ci}_{jk}} E\left[U(W^{ci}_1)|\mathcal{F}_i, P_{jk}, r\right] \tag{6}$$

subject to (3) and a short-selling constraint: $X^{ci}_{jk} \geq 0$. Here, we impose the short-selling restriction exogenously, but in the appendix we consider a simple extensioninwhichagentsendogenouslyrefrainfromsellingshort. Specifically, we allow the swindler to manipulate earnings—in particular, to report higher earnings than actual—which exposes anyone shorting a fraudulent firm to the risk of large losses. We relegate the details to Online Appendix C, and for the rest of the paper we simply exclude short sales.

The investor conditions on her own information set $\mathcal{F}_i$ (i.e., on her signals about every security), as well as on the prices of all securities in all markets.

The problem of the swindler is similar to the one of the common investor. The difference stems from the fact that the swindler is endowed with, and thereforenaturallymaytrade,anonzerofractionofthesharesinaparticular firm, namely hers. As a consequence, the swindler's trading impacts the price of her stock, and she takes this impact into account. Similar to a common investor, the swindler who owns firm l in location i solves

$$\max_{B^{sil}, X^{sil}_{jk}, S^{il}} E\left[U(W^{sil}_1)|\mathcal{F}_{il}, P_{jk}, r\right] \tag{7}$$

subject to the budget constraint (4) and X

$_{ik}$ for all risky assets, asset demands and bond holdings expressed by all investors in all locations, such that (1) markets for all securities clear; (2) risky-asset and bond holdings, $\{X^{ci}_{jk}, B^{ci}\}$, are optimal for regular investors in all locations given prices and the investors' expectations; (3) bond holdings B^{sil} and asset holdings for all securities X^{sil}_{jk} and S^{il} are optimal for swindlers given their expectations; and (4) all investors update their beliefs about the type of stock k in location j by using all available information to them—prices, interest rate, and private signals—and Bayes' rule, whenever possible.

Our equilibrium concept contains elements of both a rational expectations equilibrium and a Bayes-Nash equilibrium. All investors make rational inferences about the type of each security based on their signals, the equilibrium prices, and the interest rate, by using Bayes' rule and taking the optimal actions of all other investors (regular and swindlers) in all locations as given. The continuum of regular investors are price takers in all markets.

Swindlers, however, are endowed with the shares of a fraudulent company and take into account the impact of their trades on the share price. In formulating a demand for their security, swindlers have to consider how different prices might affect the perceptions of other investors about the type of their security. As is standard, Bayes' rule disciplines investors' beliefs only for demand realizations that are observed in equilibrium. As is usual in a Bayes-Nash equilibrium, there is freedom in specifying how out-of-equilibrium prices affect investor posterior distributions of security types.

While agents in our model condition on the observed price, allowing them to also condition on the float (i.e., the number of shares that are traded in equilibrium) would not affect our results. As long as we maintain the assumption of anonymity in trading, a swindler can simply put all her shares up for sale and simultaneously submit a demand function to make sure the market for her company clears at the price P_j. The supply of shares for both a fraudulent and a regular firm is normalized to one, so the float would be the same.

By Walras' law, we need to normalize the price in one market. We abstract from consumption at time zero for parsimony, and we normalize the price of the bond to be unity ($r=0$).

1.6 Tax equivalence

While our economy is seemingly complex, its equilibrium outcomes coincide with those of a much simpler Walrasian economy featuring distortionary and redistributive taxes. The intuition behind this result is quite straightforward: conditional on investing in a location, investors optimally invest equal amounts in all assets for which they have positive signals and in no others (the only exception is the swindler investing in her firm), but the signal is imperfect. The failure rate of the signal translates into a lower payoff relative to that obtained by a local, perfectly informed investor; the proportional loss can be thought of as a tax rate, which depends on both the investor and the target location of the investment. In addition, as long as prices are positive, which we will assume throughout,[15] swindlers have strict incentives to invest in their own firms so as to render them indistinguishable from regular firms, by submitting elastic demands at the prevailing price of all other assets in the location. This ensures a pooling equilibrium that justifies the behavior of the other investors.

The characterization of the equilibrium is particularly simple in the case where investors have constant absolute risk aversion (CARA), that is, when $U(x) = -e^{-\gamma x}$ for $\gamma > 0$. In this case there are no wealth effects on the optimal number of risky assets, and therefore common investors and swindlers in the is sometimes convenient, so we state in the appendix the straightforward generalization of Theorem 1 to any utility function U (Theorem 1b). An additional implication of this extra generality is that the conclusions of Section 2.4 on the theoretical merits of style analysis hold independently of preference or distribution assumptions. Finally, while the focus of this paper is theoretical rather than quantitative, we make use of Theorem 1b to compare the quantitative results of our asymmetric-information model with CARA and CRRA preferences. These results are similar, as the numerical Example 1 in Appendix B illustrates.

Theorem 1. Suppose that $U(x) = -e^{-\gamma x}$. There exists an equilibrium of the original economy in which the prices of all assets in each location are equal. Furthermore, the prices P_j and aggregate positions X_j^i taken by investors located in market i when investing in market j, excluding swindlers' positions in their own firms, are given as a solution to the problem

$$X^i \in \arg\max_{X \geq 0} \mathrm{E}\left[U\left(\sum_{j=1}^{K} \big((1-f_{ij})D_j - P_j \big) X_j \right) \right] \tag{8}$$

$$\kappa = \sum_{i=1}^{K} (1-f_{ij}) X_j^i, \tag{9}$$

assuming that this solution is characterized by $P_j \geq 0$.

Equation (8) formalizes the decision problem of an investor facing taxes f_{ij}, as explained above. Equation (9) is the market-clearing equation for regular firms. The left-hand side, κ, equals the supply of firms: only κ of the firms are regular. The right-hand side represents the demand for regular firms, and it depends on the tax rates: a proportion f_{ij} of the demand X_j^i is directed to fraudulent firms, leaving only the remainder to acquire regular firms. (We note that in a pooling equilibrium the swindler submits an elastic demand for her own firm, that is, absorbs the residual demand for her own firm at the price P_j, so that the market for fraudulent firms clears by construction.)

An obvious implication of Theorem 1 is that investors have an incentive to place a larger fraction of their wealth in locations where they are faced with lower implicit taxes. Indeed, if the effective taxes are sufficiently severe compared to the diversification benefit, then the investors may choose to concentrate their portfolio in a subset of locations, placing zero weights in the others.

Figure 3 provides an illustration of the trade-off between diversification and information-tax avoidance. In a symmetric setup, the higher the correlation between any two locations, the lower the threshold for f_{ij} above which agent i does not wish to invest in market j, and therefore the fewer markets the investor

In words, the aggregate dividends κD_j in location j are all paid to investors in proportion to their holdings of regular firms in this location, and no dividend gets lost.

Theorem 1 provides a micro-foundation to the common practice (especially in international economics, but also more broadly) of using taxes (or "wedges") as a reduced-form way of modeling informational frictions, as long as these taxes are redistributive, rather than iceberg costs. Theorem 1b in the appendix shows that the equivalence between this information-asymmetry model and a simple, Walrasian economy with distortionary, redistributive taxes holds for any concave preferences and distributional assumptions on dividends.

For the purposes of the remainder of the paper, Theorem 1 makes the description of an equilibrium and its properties relatively easy, a feature that we use in Section 2. In addition, it provides an intuitive analogy between informational disadvantages and taxes.

1.7 Further discussion of assumptions and robustness

To conclude this section, we make a few remarks on the generality of the model. In particular, after a brief discussion of the concept of "swindlers," we concentrate on describing how the model can accommodate repeated trading and information revelation over time.

1.7.1 Swindlers. From a modeling perspective, swindlers prevent revelation through the price. Thus, while strategic and well informed, they end up playing a similar role to noise traders (or agents with random endowments) in a rational expectations equilibrium. In terms of interpretation, a literal, but somewhat narrow, real-world counterpart to the activity of swindlers inside the model would be corporate fraud. Based on SEC enforcement cases, identified (and prosecuted) cases of fraud pose a nonnegligible source of losses to common shareholders, which are around 0.42% of market capitalization on an annual basis.[16] This number likely substantially understates the true extent of damages suffered by common investors, because the number of frauds associated with SEC enforcement action is small compared to the substantive number of cases adjudicated by class actions or the ones never identified.[17] Thus, while outright

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[16] Using data from the latest available decennial report on fraudulent financial reporting covering the years 1998–2007, the Committee of Sponsoring Organizations of the Treadway Commission (COSO) presents data on 347 cases of identified fraud. The most prevalent cases are Enron and Worldcom over that period, but the sample contains several other nontrivial cases of fraud. The study reports that the average shareholder value of the identified firms is slightly larger than one billion, implying that the aggregate shareholder value of the affected firms is about 347 billion. The vast majority of the firms either go bankrupt or are involuntarily delisted in the aftermath of fraud. The sample does not include any major recession (other than the small recession of 2001). Including data for the 2008 recession is particularly informative, since the economic weakness unveiled two further major scandals (Madoff 65 billion and the Lehman accounting scandal, 50 billion), bringing the market value of the capitalization affected by fraud to 462 billion or approximately 42 billion per year. This amounts to 42 basis points of the stock market capitalization at the beginning of the sample.

[17] In a *Forbes* article, James Kaplan, cofounder and chairman of Audit Integrity notes: "The 347 companies prosecuted in the decade through 2007 represent a small fraction of the number of financial fraud cases that

fraud is part of the motivation for introducing swindlers, in the real world several actions of insiders may not fit the strict description of fraud, yet result in losses for common investors. For this reason, we favor a broader interpretation of our swindlers as insiders who possess and act strategically on superior information.

The baseline model assumes that fraudulent firms produce zero output, but that idealization is not necessary. Making fraudulent firms valuable or risky impacts the swindlers' incentives to retain ownership in order to pool, but a pooling equilibrium continues to exist, and an appropriate version of Theorem 1 continues to hold. (An illustration of this statement, in an even richer environment, is provided by the multiperiod extension described in the next section.) A pooling result would also hold if there were multiple types of regular firms in a location, an issue that we address in Online Appendix B.

1.7.2 Information revelation and repeated trade. In the baseline model, all trade takes place in period 0 and all uncertainty is resolved in period 1. The question arises whether a pooling equilibrium survives type revelation as cashflows are realized over time. Succinctly put, the answer is yes.

To address this question, in the Online Appendix A we present a minimal model extension that features sequential trading with investors updating their information prior to each round of retrade. Here, we only summarize the new economic issues that arise in this extension and refer the reader to the Online Appendix for the details of the setup and the precise statements of the propositions. We also further discuss assumptions and alternatives.

Specifically, we consider the same static model as in the paper, except that there are three periods rather than two. (The logic extends to more than three periods.) Agents trade in periods 0 and 1 and consume in periods 1 and 2. All firms in a location pay a location-specific dividend in period 1. Only regular firms pay a dividend at time 2. Importantly, some (but not all) of the fraudulent firms become publicly identified as such ("go bankrupt") before date-1 trading commences.

This extension of the baseline model features two new elements: (a) the time-1 asset endowments (i.e., asset ownership before trading) are different from the time-0 endowments, and (b) due to the assumption that some fraudulent firms become publicly known in the intermediate period, there is updating (but not full learning) in period 1 prior to trading.

Under appropriate parametric conditions, a shadow-tax equilibrium akin to the static one (Theorem 1) characterizes both periods of trading. The argument becomes more complex in a dynamic framework because of two issues. First, because fraudulent firms pay a positive dividend in period 1, the swindler faces a trade-off in period 0 between retaining a fraction of her shares and selling the other ones for a higher price, and selling all her shares at a lower (revealing)

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occurred. Very few frauds result in SEC enforcement action; many more are adjudicated by class actions. Most are recorded only in stakeholder disappointment, large price drops, bond defaults and insolvency" (Kaplan 2010).

price. Second, swindlers (those whose firms do not go bankrupt in period 1) must choose to pool also in period 1, a condition that comes down to the aggregate demand for any swindler's firm from the other agents being higher at date 1 than at date 0. That way, the swindler is a net seller of her own firm shares, in line with her incentives given that her firm is over-priced in a pooling equilibrium.

2. Informationally Inefficient Markets: Implications

In this section we exploit the equivalence formalized in Theorem 1 between informational frictions and taxes to study the ability of popular performance-evaluation approaches to appropriately identify investors with "skill," that is, investors who select stocks based on informative signals. Throughout we envisage an econometrician, by definition uninformed, who observes the return obtained by an investor on her portfolio and is trying to infer if that investor had valuable signals in choosing her portfolio.

The first question we address (Sections 2.1–2.2) is whether CAPM alphas provide an appropriate measure of an investor's informational advantage. Specifically, in these two sections we assume that dividends are joint normal so that the CAPM would hold in the absence of informational asymmetries. We also assume that investors have CARA preferences, so that we can provide simple, closed-form solutions for equilibrium prices. Using these prices, in Sections 2.2 and 2.3 we analyze the properties of equilibrium alphas inside the model and conclude that they are problematic: investors with no skill may have positive alpha, while investors with skill may have negative alpha. This shows that even though in our model the only skill is a stock selection skill, the CAPM alphas do not provide an appropriate measure of this skill.

Motivated by the negative results of Sections 2.1–2.3, in Sections 2.4 and 2.5 we analyze the essentially unique, meaningful performance measure in our model. This performance measure, whose validity in our model is independent of return or preference specifications, is closely related to W. Sharpe's style analysis.

2.1 Equilibrium prices

To ensure that the CAPM would hold in the absence of informational frictions, in Sections 2.1 and 2.2 we assume that the dividends D_j are jointly normal. For simplicity we also assume that they have the same mean, which we normalize to unity. To obtain explicit expressions for equilibrium prices, we endow investors with CARA utilities, $U(W) = -e^{-\gamma W}$.

We let $\lambda_{ij} \geq 0$ denote the Lagrange multiplier associated with $X_j^i \geq 0$, and define

$$p_{ij} \equiv 1 - f_{ij} \tag{10}$$

as the effective payoff to investing in assets of location j. Note that p_{ij} is the probability that security j is regular given that the signal of investor i

identifies it as such. Clearly, $p_{ij} \geq \kappa$, with strict inequality if the investor's signal is valuable. Given the CARA-normal setup, the first-order condition of an investor in location i faced with problem (8) is

$$\gamma \operatorname{cov}\left(p_{ij} D_j, \sum_{k=1}^{K} p_{ik} D_k X_k^i\right) = p_{ij} - P_j + \lambda_{ij}. \tag{11}$$

Dividing this equation by p_{ij} and summing over all agents i yields

$$\gamma \operatorname{cov}\left(D_j, \kappa D^a\right) = 1 - \frac{P_j}{K} \sum_{i=1}^{K} p_{ij}^{-1} + \frac{1}{K} \sum_{i=1}^{K} p_{ij}^{-1} \lambda_{ij}, \tag{12}$$

where we introduced the notation D^a for the average dividend, $D^a \equiv \frac{1}{K}\sum_{j=1}^{K} D_j$, and used the fact that (9) and exchanging the order of the summation yield

$$\sum_{i=1}^{K}\sum_{k=1}^{K} p_{ik} D_k X_k^i = \sum_{k=1}^{K} D_k \sum_{i=1}^{K} p_{ik} X_k^i = \kappa K D^a. \tag{13}$$

These calculations lead to the following result.

Proposition 1. The price P_j is expressed as

$$\begin{aligned} P_j &= \left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}^{-1}\right)^{-1} \times \left(1 - \gamma \operatorname{cov}\left(D_j, \kappa D^a\right) + \frac{1}{K}\sum_{i=1}^{K} \lambda_{ij} p_{ij}^{-1}\right) \\ &= \left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}\right) \times \left(1 - \gamma \operatorname{cov}\left(D_j, \kappa D^a\right) + \frac{1}{K}\sum_{i=1}^{K} \lambda_{ij} p_{ij}^{-1}\right) \times \frac{\left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}^{-1}\right)^{-1}}{\left(\frac{1}{K}\sum_{i=1}^{K} p_{ij}\right)}. \end{aligned} \tag{14}$$

The proposition provides a natural formula. In Equation (14), the first term captures the average post-tax payoff to investors, the second the risk adjustment and the effect of the shorting constraint, while the third measures dispersion in p_{ij} across agents. Equation (14) shows that two asset classes may be priced differently even when containing the same amount of aggregate risk (that is, $\operatorname{cov}(D_j, D^a)$ is the same for all j) and being held in positive amounts by all agents ($\lambda_{ij} = 0$). As long as $p_{ij} \neq p_{ij'}$ for some i for two asset classes j and j', it is possible that $P_j \neq P_{j'}$. This observation will prove useful in the next section.

2.2 Alpha does not measure skill

Next, we obtain some implications of the model for CAPM alphas. By CAPM alphas we mean the estimates of the constant in a regression of the excess return obtained by an investment strategy on the excess return of the market portfolio. Throughout the paper, we do not concern ourselves with estimation issues. We exclusively focus on the implications of our theory for the moments of such regressions.

To start, we define R_j^p as the gross return of a passive (or index) return in location j. This is the gross return obtained by simply buying all the firms in location j. (This would be the return of an uninformed investor, who doesn't have access to any private signals.) Given the assumptions of the model, this return is given by $R_j^p = \frac{\kappa D_j}{P_j}$, with expectation $\frac{\kappa}{P_j}$. Similarly, define the average price $P^a \equiv \frac{1}{K}\sum_{k=1}^K P_k$, and the return on an index replicating the market portfolio is $R^a = \frac{\kappa D^a}{P^a}$. Recalling that the interest rate is normalized to zero, we define α_j as the constant in the regression of the observed (passive) return of the index in location j on the market portfolio return:

$$R_j^p - 1 = \alpha_j + \beta_j^p (R^a - 1) + \varepsilon_j^p. \tag{15}$$

We have the following result.

Proposition 2. The passive alpha with respect to the market equals

$$\alpha_j = \left(\beta_j^D \frac{P^a}{P_j} - 1\right) + \frac{\kappa}{P_j}\left(1 - \beta_j^D\right), \tag{16}$$

where β_j^D is the "cash-flow beta"

$$\beta_j^D = \frac{\text{cov}(D_j, D^a)}{\text{var}(D^a)}. \tag{17}$$

Note that in the special case in which there is no asymmetric information ($p_{ij} = \kappa$) and all positions are strictly positive ($\lambda_{ij} = 0 \ \forall i, j$) Equations (14) and (16) imply the usual CAPM relation ($\alpha_j = 0$).[18]

However, in the presence of informational asymmetries, α_j is nonzero in general, even for passive strategies. To see this in the simplest possible case, consider a world with $\beta_j^D = 1$ for all j. Accordingly,

$$\alpha_j = \frac{P^a}{P_j} - 1 = \frac{\frac{1}{K}\sum_{j=1}^K \left(\frac{1}{K}\sum_{i=1}^K p_{ij}^{-1}\right)^{-1}}{\left(\frac{1}{K}\sum_{i=1}^K p_{ij}^{-1}\right)^{-1}} - 1. \tag{18}$$

The above equation implies that some asset classes may still exhibit prices that are lower (or higher) than average, despite all assets having the same exposure

[18] To see this, notice that Equation (14) implies that $P_j = \kappa - \gamma\kappa^2\beta_j^D\sigma_a^2$. Then it follows from (16) that $\alpha = \beta_j^D \frac{\kappa - \gamma\kappa^2\sigma_a^2}{P_j} - 1 + \frac{\kappa}{P_j}\left(1 - \beta_j^D\right) = \frac{\kappa - \gamma\kappa^2\beta_j^D\sigma_a^2}{P_j} - 1 = 0.$

to aggregate risk and the same expected dividend. For instance, a lower overall quality of information in asset class j (low values of p_{ij} compared to other asset classes) translates into a lower-than-average price for that class; because $\alpha_j = \frac{P^a}{P_j} - 1$, even an index investment in such a class has positive alpha.

If uninformed (passive) strategies command alphas, then alphas cannot be an accurate measure of an investor's information advantage ($p_{ij} > \kappa$), which we interpret as "skill." Indeed, continuing with the assumption that $\beta_j^D = 1$ for all j, the alpha resulting from a regression of the return that an investor i obtains when investing in location j on the return of the market portfolio is given by

$$\alpha_{ij} = \frac{p_{ij}}{\kappa}\frac{P^a}{P_j} - 1. \tag{19}$$

Hence, even an investor who has an informational advantage might exhibit a negative alpha when that informational advantage happens to be in an asset class that is comparatively more expensive than the average asset class, that is, $P^a < P_j$.[19]

Equations (18) and (19) imply that when $\beta_j^D = 1$ both passive and active alphas depend exclusively on p_{ij} and κ—neither on risk aversion, nor on the volatility of dividends. Equilibrium portfolio allocations and expected returns on the other hands do depend on all the parameters of the model. This property implies that the magnitude of alphas and the magnitude of portfolio biases are not linked in this model, as the numerical Example 2 in Appendix B illustrates.

The reason the CAPM fails to assign zero alpha, even to passive strategies, is qualitatively different from the arguments that have been proposed so far. Unlike elsewhere in the literature, in our setup investors don't possess any signals on the realization of D_j, so they are on equal footing about predicting the return of an asset class. It is tempting to attribute the failure of the CAPM in our model to the fact that different investors hold different mean-variance efficient portfolios, so that the market portfolio is not mean-variance efficient for any investor. This fact, however, is not sufficient to render the CAPM alpha an inaccurate measure of skill: Suppose, for instance that all prices across all asset classes are equal ($P_j = P$), which would occur for instance if the informational advantages are symmetric ($p_{ij} = p$ for all $i \neq j$ and some positive $p < 1$), and all betas are unity. In that case investors still choose different mean-variance efficient portfolios, depending on their locations. Yet, Equation (16) shows that alphas are zero for passive strategies, whereas Equation (19) shows that informed investors have positive alphas.

What makes alpha a valid measure of performance in this special case? As we show in more generality in Section 2.5, the key feature of this special case

[19] From Equation (19), knowledge of class-by-class alphas (or returns) is sufficient to compare the level of information about a certain class across investors. However, if the econometrician only observes the overall return of an investor (or simply uses a coarser definition of asset classes), and assuming that different investors invest in different asset classes, then the equation does not rank investors by informational advantage.

is that the market portfolio is mean-variance efficient from the perspective of an *uninformed* investor. However, this property is special to this example. In general the market portfolio is not mean-variance efficient even from the perspective of an uninformed agent, and hence the CAPM alpha is not a valid measure of performance.

We conclude this section with a parenthetical remark on the magnitude of passive alphas implied by equations (16) and (18). Inspection of Equation (18) shows that when $\beta_j^D = 1$ for all assets j, the alphas of passive strategies are bounded above by $\kappa^{-1} - 1$ and below by $\kappa - 1$. However, when β_j^D varies across asset classes, then the passive strategy alphas need not obey these bounds, and the exact magnitude of the passive strategy alphas does not only depend on informational asymmetry assumptions (specifically on the assumed values of p_{ij} and κ) but also on statistical assumptions about the second moment of D. Example 3 in Appendix B illustrates this point.

2.3 Negative alpha for an investor's optimal portfolio

In the previous section we showed how CAPM alphas can be positive for uninformed passive returns and negative for the returns obtained in a specific asset class by an investor possessing selection skill in that class.

In this section we examine the alpha of an investor's optimally chosen portfolio, rather than her return within an individual asset class. In two important papers, Mayers and Rice (1979) and Dybvig and Ross (1985) have shown that a mean-variance efficient portfolio utilizing useful private information has a positive alpha with respect to any reference portfolio as long as the informational advantage helps the investor choose individual assets better ("selection ability"), but without allowing her to predict the return of the benchmark portfolio any better than other market participants ("no timing ability"). While in our framework investors have pure selection ability in the sense of Mayers and Rice (1979) and Dybvig and Ross (1985), their results no longer apply. Indeed, in this subsection we show that the alpha of an informed investor's optimal portfolio may be negative, even though she possesses stock selection skill.

To better compare our results with the literature, we will not only consider alphas with respect to the market portfolio return (CAPM alphas), but we will allow for an arbitrary benchmark or reference portfolio w^B of risky asset class weights.

To relate to the literature, we present a sufficient condition that allows extending the results of Mayers and Rice (1979) and Dybvig and Ross (1985) to our framework. To that end, let R^I denote the gross return obtained by a (potentially informed) investor on her entire portfolio and R^B the gross return on an uninformed portfolio assigning weights w^B to the various asset classes. Denote by $E(.)$ expectations under the econometrician's information set and by σ^X the standard deviation of R^X, $X \in \{I, B\}$, under the same information set.

Proposition 3. Suppose that

$$\frac{|E(R^B)-1|}{\sigma^B} < \frac{E(R^I)-1}{\sigma^I}, \tag{20}$$

Then the expected alpha of a regression of the excess return of the investor's return on the excess return of the benchmark portfolio is positive.

In words, Proposition 3 states that if the absolute value of the Sharpe ratio of the benchmark portfolio is smaller than the Sharpe ratio of the investor's optimal portfolio, then the investor's portfolio exhibits positive alpha with respect to the benchmark return.

In Dybvig and Ross (1985) condition (20) is automatically implied by the optimality of the investor's portfolio: A mean variance investor with the ability to short will always choose a portfolio with an absolute Sharpe ratio (weakly) larger than any given portfolio w^B. Indeed, both w^B and $-w^B$ are feasible portfolios, even though they involve ignoring one's private information. Hence, in a mean-variance framework, the absolute value of the Sharpe ratio of the reference portfolio cannot exceed the Sharpe ratio of her optimally chosen portfolio (by revealed preference).

However, without shorting, condition (20) may fail, and in fact even if the reference portfolio involves *only positive* weights: Intuitively, in order to be able to attain the *absolute value* of the Sharpe ratio of the reference portfolio, an investor may need to be able to invest in both w^B and $-w^B$, which is not the case in our framework.

Allowing for the violation of condition (20), it is possible to produce examples where the alpha of an informed investor on her entire portfolio is negative. In Appendix B (Example 4), we provide a simple numerical example in which an informed investor has a negative alpha with respect to the market portfolio.

We note parenthetically that the failure of condition (20) is not special to the presence of shorting frictions. Any portfolio friction that could render the constrained optimal portfolio mean-variance inefficient could lead to a failure of condition (20). In Appendix D (Example 1) we illustrate this with an example featuring a borrowing constraint. We show that an informed agent may obtain a negative alpha on her (constrained optimal) portfolio even in situations in which all weights of the reference portfolio, all portfolio weights of all agents, and all expected excess returns are positive.

To summarize, once we allow for portfolio constraints, the correspondence between alpha and skill—even at the level of an investor's total portfolio return—may no longer hold. This is particularly problematic in practice, because an econometrician typically does not know whether an investor's observed portfolio return results from an interior or a constrained optimal portfolio, and therefore cannot control appropriately for this issue.

2.4 General properties of evaluation measures and style alphas

The previous section shows that the CAPM fails to assign zero alpha to passive strategies. We show here that this failure is responsible for the imperfect mapping between skill and alpha that we highlighted above. In particular, we show that assigning zero alpha to passive strategies is actually a sufficient condition for a performance measure to be valid (in the sense of correctly identifying a skilled investor). Moreover, this validity result is independent of whether the investor's portfolio choice is interior or constrained optimal across asset classes. It is also independent of preference or return distribution assumptions, because it is only based on tax equivalence, which holds generally (Theorem 1b). As a practical illustration of the results, we show that the style alpha measure proposed by W. Sharpe is a valid performance measure. We start with a definition.

Definition 1. Let g be a functional mapping random variables into the space of real numbers such that

1. Letting $R_j^{e,p}$ denote the excess passive return in location j, $g(R_j^{e,p})=0$.
2. g is linear, that is, for two random variables X and Y and a scalar A, $g(X+Y)=g(X)+g(Y)$ and $g(AX)=Ag(X)$.
3. $g(1)>0$.

If such a functional g exists we will refer to it as a "valid performance functional."

The three requirements listed in Definition 1 are intuitive. We require that g assign the value zero to all passive (uninformed) excessive returns (property 1) and all portfolios thereof (property 2). The third property states that a riskless excess return should be assigned a positive value.

Assuming the existence of a valid performance functional in the sense of Definition 1, we next show that it correctly identifies an investor's informational advantage. To see this, note that the excess return of an informed investor i in our model can be written as

$$R^{e,i}=\sum_{j=1}^{K}\left(q_{ij}R_j-1\right)w_j^i,$$

where $q_{ij}\equiv\frac{p_{ij}}{\kappa}\geq 1$ and $w_j^i\geq 0$ is the portfolio weight of agent i represented by asset class j. Hence,

$$g\left(R^{e,i}\right)=g\left(\sum_{j=1}^{K}q_{ij}\left(R_j-1\right)w_j^i+\sum_{j=1}^{K}\left(q_{ij}-1\right)w_j^i\right)$$

$$= 0 + g(1) \sum_{j=1}^{K} \left(q_{ij} - 1\right) w_j^i \tag{21}$$

$$\geq 0.$$

We summarize the implications of the above discussion in the following proposition.

Proposition 4. Assume that a valid performance functional g exists and fix an investor i. Then $g\left(R^{e,i}\right) > 0$ if and only if investor i has an informational advantage ($q_{ij} \equiv \frac{p_{ij}}{\kappa} > 1$) in at least one asset class, where she assigns a positive weight $w_j^i > 0$.

Remark 1. We note that Equation (21) also shows that the functional g is essentially unique: any two measures g and g' differ at most by a multiplicative constant.

One way to construct a functional g is the so-called "style" analysis, proposed by Sharpe (1992). According to this approach, the return of each manager is regressed on the passive returns of all possible asset classes. Moreover, to interpret the betas as portfolio weights, one additionally requires that the betas on the passive strategies add up to one. (In practice, they are also restricted to be positive, to satisfy the no-shorting constraints faced by mutual-fund managers.) The constant (alpha) of such a regression is interpreted as a manager's skill.

Viewing style analysis as mapping the (excess) return of a manager to a value of alpha, it is straightforward to show that it satisfies all the aforementioned properties of the functional g. We record the result formally:

Proposition 5. Let $w_j^i = \frac{P_j X_j^i}{W_0^{ci}}$ be the portfolio weight of the investment in location j by an investor in location i. Consider the style regression of the gross return obtained by such an investor on the passive returns, including the risk-free one. The constant α_i^s in this regression is the portfolio-weighted informational advantage of investor i across all markets in which she invests:

$$\alpha_i^s = \sum_{j=1}^{K} \left(\frac{p_{ij}}{\kappa} - 1\right) w_j^i. \tag{22}$$

An alternative way of formulating the functional g is as follows. Let Σ denote the covariance matrix of passive excess returns $R_j^{e,p}$, $E(R^{e,p})$ the vector of expected excess passive returns, $w = \Sigma^{-1} E(R^{e,p})$ a mean-variance efficient portfolio from the perspective of an uninformed econometrician, and $R^{MVE} = w^\top R^{e,p}$ the excess return of the portfolio. Then the functional

$g(R^e) \equiv E(R^e) - \text{cov}(R^e, R^{MVE})$ satisfies all the requirements of the functional g, because it is linear, satisfies $g(1)=1$, and most importantly assigns the value zero to all passive excess returns. This observation formalizes the claim we made in Section 2.2: the reason for the inadequacy of CAPM alphas is that the market portfolio is not mean-variance efficient even from the perspective of an uninformed econometrician. (See also Ferson and Siegel (2001) for the properties of unconditionally efficient portfolios for the purposes of performance evaluation.)

Our analysis in this section is related to Chen and Knez (1996), who characterize performance measures satisfying a reasonable minimal set of requirements in a general payoff-and-information environment. Our special model structure implies a tighter characterization—essentially, our performance measure is unique, assigns positive alpha to informed strategies, and it can be thought of as a style alpha.

2.5 Investor-specific performance evaluation

A key requirement for a valid functional g is that it assign zero alpha to passive strategies. An issue that we did not address in the previous section is that the requirement need only apply with respect to the locations in which a given investor i participates. Indeed, Equation (21) continues to hold even if the values $g(R_j^{e,p})$ are set arbitrarily whenever the investor chooses $w_j^i = 0$.

This observation is of practical importance because in the real world many portfolios are concentrated in only a few asset classes, and virtually all shun some asset classes. It also helps explain the widespread use of heterogeneous benchmarks. Thus, if the goal is to evaluate the stock-picking skills of an asset manager who only invests in, say, Finnish stocks, then our analysis provides a justification for regressing her return only on the Finnish stock market index rather than some global index, or a set of indices from several countries. We also note that adding more classes not only does not help, but in fact hurts by deteriorating the quality of estimation and inference with finite data.

The above discussion helps us illustrate an additional point of some theoretical interest: One can find valid, investor-specific g_i even when a functional g pricing all passive strategies does not exist. The easiest way to illustrate this point is by using a minimal example whereby an equilibrium features an unexploitable arbitrage. For instance, consider an economy in which *(a)* the passive portfolios in two locations (say, locations j and j') have the same dividends from the perspective of a passive investor ($\kappa D_j = \kappa D_{j'}$) but different prices ($P_j \neq P_{j'}$);[20] *(b)* investor j invests only in market j, because $\frac{p_{jj}}{P_j} > \frac{p_{jj'}}{P_{j'}}$ and similarly investor j' only invests in market j'. The absence of shorting makes this arbitrage opportunity compatible with equilibrium. A global performance

[20] This could occur in equilibrium, for instance, because the investors in a third location j'' are better informed about one of these two locations, resulting in a higher price for its securities.

functional g applying simultaneously to $R_j^{e,p}$ and $R_{j'}^{e,p}$ does not exist, yet investor-specific performance functionals g_j and $g_{j'}$ are easy to construct, for example, by regressing each investor's return on the passive returns in the asset classes in which she invests.

2.6 Summing alphas

We now take a closer look at the cross-section of portfolio performance in our model. The starting point is the general observation that, relative to the market, the average alpha must be zero by construction. However, in our model all the investors are assumed to have some information, which should allow them to improve on the market portfolio and consequently exhibit positive alphas.

To discuss this issue, we revisit Section 2.2 and concentrate on an economy that is symmetric with respect to the various locations. In this economy, $P_j = P = P^a$, and Equation (19) gives $\alpha_{ij} = \frac{p_{ij}}{\kappa} - 1 > 0$. All individual alphas are positive, so the portfolio-weighted average of the alphas (across investors) appears to be strictly positive. This conclusion is not correct, though, because the analysis so far has ignored the swindlers' investment in their own firms. Indeed, these agents invest a nonzero fraction of their portfolio in an asset costing $P > 0$ and paying back zero, that is, offering a net return of -100%.

One can see explicitly the negative return to the swindlers' retained holdings in their own firms in the market-clearing equation from Theorem 1. Focusing on a single market, recalling that $1 - f_{ij} = p_{ij}$, and summing across investors i expresses Equation (9) as

$$0 = \sum_{i=1}^{K} \left(\frac{p_{ij}}{\kappa} - 1 \right) X_j^i + (-1) \times \left(1 - \sum_{i=1}^{K} X_j^i \right). \tag{23}$$

The right-hand side of Equation (23) contains two terms. The first term is positive and captures the intuition of aggregate positive alphas. The second term, though, is negative, because $\sum_{i=1}^{K} X_j^i < 1$: the difference $1 - \sum_{i=1}^{K} X_j^i$ represents the swindlers' position in their own firms in location j, and -1 is the associated net return.

The alphas realized by the swindlers combine the -100% on their own firms with the positive values on the rest of their portfolios, but are negative in the aggregate. This is despite the fact that the swindlers possess superior information. Given their endowment of worthless stock, swindlers are actually better off retaining some of their shares in their effort to pool with the regular stock. The reason for their negative alpha is not suboptimal behavior, but rather the nature of their initial endowment.[21]

[21] This phenomenon is related to discussions in Kacperczyk et al. (2014, 2016).

3. Conclusion

We develop a multiple-market, multiple-investor model, whereby informational asymmetries act as distortionary and redistributive capital taxes. By explicitly modeling the incentive to diversify across asset classes, and introducing strategic trading considerations for some traders, we can dispense with noise trading, yet keep prices nonrevealing. Moreover, the duality between the model and a tax economy makes the model quite tractable to analyze, without requiring CARA utilities and normal dividends.

By drawing a distinction between asset classes (sets of assets that appear identical from the perspective of an uninformed agent) and individual assets within asset classes, the model can account for portfolio biases toward specific asset classes for any realization of the signals about the quality of individual assets. Hence the model provides a simple and analytically convenient framework to model persistent portfolio biases toward a set of asset classes, underdiversification, and portfolios with noninterior (zero) holdings of individual assets.

To illustrate the analytical tractability of the model, we revisit an established literature that analyzes the properties of popular performance evaluation measures. Our framework allows a particularly clean distinction between pure selection and timing abilities. Without trivializing the possible importance of timing information, we show that the specific informational assumptions we adopt provide a simple and intuitive theoretical basis for portfolio evaluation criteria such as style analysis and fund-dependent choice of benchmarks, which are widely used in practice.

Appendix

A. Proofs

Proof of Theorem 1. This theorem is a special case of the more general Theorem 1b, stated and proved below. In particular, Equations (8) and (9) in the statement of Theorem 1 constitute a particular case of the system (A.1)–(A.4), because with CARA preferences all objectives are independent of the wealth endowment and therefore identical, and consequently so are the portfolios. ■

To state the result for a general utility function U, let X_j^{ci} be the per capita number of shares invested by a common investor from location i in assets in location j, and X_j^{si} the analogous number of shares invested by a swindler in location i in all firms other than his own. We let T^i be the mass of shares sold by a swindler in his own firm.

Theorem 1b. There exists an equilibrium of the original economy in which the prices of all assets in each location are equal. Furthermore, the prices P_j and equilibrium positions solve the system

$$X^{ci} \in \arg\max_{X\geq 0} \mathrm{E}\left[U\left(\sum_{j=1}^{K}\big((1-f_{ij})D_j-P_j\big)X_j+P_i\right)\right] \tag{A.1}$$

$$X^{si} \in \arg\max_{X\geq 0} \mathrm{E}\left[U\left(\sum_{j=1}^{K}\big((1-f_{ij})D_j - P_j\big)X_j + T^i P_i\right)\right] \tag{A.2}$$

$$T^i = \sum_{j\neq i} \frac{f_{ji}}{1-\kappa}\left(\kappa X_i^{cj} + (1-\kappa)X_i^{sj}\right) \tag{A.3}$$

$$\kappa = \sum_{i=1}^{K}(1-f_{ij})\left(\kappa X_j^{ci} + (1-\kappa)X_j^{si}\right), \tag{A.4}$$

assuming that this solution is characterized by $P_j \geq 0$.

Equation (A.1) is the obvious objective for a common investor. Equation (A.2) is the objective of a typical swindler in location i. Unlike a common investor, who is endowed with wealth P_i, this investor's wealth consists of the number of shares that he trades (sells) in equilibrium, T^i. This number of shares equals the number of shares bought by investors, both common ones and swindlers, who, in other locations, received positive signals, as captured by Equation (A.3). The conditional independence of the signals allows the simplification that the demand is the same for all fraudulent firms. In this equation, the term $\frac{f_{ji}}{1-\kappa}$ on the right-hand side represents the proportion f_{ji} of the demand from location j to location i being directed to fraudulent firms, and shared among the $1-\kappa$ firms there. Finally, Equation (A.4) is the market-clearing condition for a common firm in location j.

Proof of Theorem 1b. We start with an equilibrium in the simplified competitive (symmetric information) tax economy, and then proceed through a couple of steps. First, we construct demand curves in the original economy, making use of the tax-economy equilibrium. Second, we check that these demand curves are optimal given the other agents', and the markets clear.

We specify the demands of the agents using the solution $(X_j^{ci}, X_j^{sil}, P_j)$ to (A.1)–(A.4):

$$X_{jk}^{ci} = \big(1-f_{ij}\big)\kappa^{-1} X_j^{ci} \iota_{jk}^{il} \mathbf{1}_{\left(P_{jk}=P_j\right)} \tag{A.5}$$

$$X_{jk}^{sil} = \big(1-f_{ij}\big)\kappa^{-1} X_j^{sil} \iota_{jk}^{il} \mathbf{1}_{\left(P_{jk}=P_j\right)} \tag{A.6}$$

$$S^{il} = \begin{cases} [0,\infty) & \text{if} \quad P_{il} = P_i \\ 0 & \text{if} \quad P_{il} \neq P_i \end{cases}. \tag{A.7}$$

Recall that we are looking for a Nash equilibrium, in which all agents take the others' actions as given. For any agent-asset pair excluding a swindler and his own firm, this means that the price is taken as a given. A swindler can impact the price of his firm by his choice of quantity, taking into account the demand curves of all the other agents.

In words, all investors buy the same number of shares in each market as in the tax economy, but they split this position (equally) only among the firms about which they receive a good signal—note that the multiplicative factor $\big(1-f_{ij}\big)\kappa^{-1}$ equals the reciprocal of the probability that a given signal is good—as long as the price equals the pooling equilibrium price P_j. Implicitly, the agents treat any firm whose price is not P_j as a fraudulent firm, which is intuitively justified by the fact that the only possible deviation resulting in a different price is by a swindler in his own asset.[22] The swindler submits an elastic demand at P_j.

[22] Consequently an appropriate adaptation of the notions of sequential equilibrium or trembling-hand perfection would result in the demand curves (A.5) and (A.6).

It is easy to see that, given these demand curves, markets clear. To see that X_{jk}^{ci} is optimal, start by writing the expected utility for the agent as

$$\mathrm{E}\left[U\left(\sum_{j=1}^{K}\int_{k}\left(D_{jk}-P_{j}\right)X_{jk}^{ci}dk+P_{i}\right)|\iota^{il}\right] \tag{A.8}$$

$$=\mathrm{E}\left[U\left(\sum_{j=1}^{K}\int_{k}\left(\rho_{(jk)}D_{j}-P_{j}\right)X_{jk}^{ci}dk+P_{i}\right)|\iota^{il}\right]$$

and note that, by Jensen's inequality, this utility is maximized by choosing X_{jk}^{ci}, for fixed j, to be measurable with respect to ι_{jk}^{il}. In words, the agent invests identically in all assets in market j in which she received the same signal. Furthermore, the agent will not buy any asset with low signal ($\iota_{jk}^{il}=0$), because the asset returns zero for sure but has a positive price.

Take k with $\iota_{jk}^{il}=1$ and let $\hat{X}_{j}^{ci}=X_{jk}^{ci}Pr\left(\iota_{j\cdot}^{il}=1\right)=X_{jk}^{ci}\frac{\kappa}{1-f_{ij}}$. Then both sides in Equation (A.8) are also equal to

$$\mathrm{E}\left[U\left(\sum_{j=1}^{K}\left((1-f_{ij})D_{j}-P_{j}\right)\hat{X}_{j}^{ci}+P_{i}\right)\right], \tag{A.9}$$

which is the same as (A.1), so that $\hat{X}_{j}^{ci}=X_{j}^{ci}$. It follows that the optimal position is

$$X_{jk}^{ci}=Pr\left(\iota_{jk}^{il}=1\right)^{-1}\hat{X}_{j}^{ci}\iota_{jk}^{il}=(1-f_{ij})\kappa^{-1}X_{j}^{ci}\iota_{jk}^{il}. \tag{A.10}$$

Equation (A.5) is immediate.

The same argument holds for the choice that a swindler makes with respect to all assets but her own. When choosing the position in her own asset, the only consideration is the time-zero revenue $(1-S^{il})P_{il}$, because the asset pays zero. Given the other investors' demands, the insider must ensure that $P_{il}=P_{i}$. To that end she submits a demand that fails to clear the market at $P_{il}\neq P_{i}$ and is willing to take any position at $P_{il}=P_{i}$. ■

Proof of Proposition 2 By the definition of α_j,

$$\alpha_{j}=\frac{\kappa}{P_{j}}-1-\frac{cov\left(\frac{\kappa D_{j}}{P_{j}},\frac{\kappa D^{a}}{P^{a}}\right)}{\left(\frac{\kappa}{P^{a}}\right)^{2}var(D^{a})}\left(\frac{\kappa}{P^{a}}-1\right)=\frac{\kappa}{P_{j}}-1-\beta_{j}^{D}\frac{P^{a}}{P_{j}}\left(\frac{\kappa}{P^{a}}-1\right)$$

$$=\left(\beta_{j}^{D}\frac{P^{a}}{P_{j}}-1\right)+\frac{\kappa}{P_{j}}\left(1-\beta_{j}^{D}\right). \quad ■$$

Proof of Proposition 3 Let $R^{I,e}=R^{I}-1$, $R^{B,e}=R^{B}-1$, $\beta=\frac{cov\left(R^{B},R^{I}\right)}{\left(\sigma^{B}\right)^{2}}$, $\rho_{B,I}=\frac{cov\left(R^{B},R^{I}\right)}{\sigma_{B}\sigma_{I}}$, and $sgn(x)=1$ for all $x\geq 0$ and $sgn(x)=-1$ for all $x<0$. Assumption (20) implies

$$E\left(R^{I,e}\right)>\frac{\sigma^{I}}{\sigma^{B}}sgn\left(E\left(R^{B,e}\right)\right)E\left(R^{B,e}\right)=\frac{sgn\left(E\left(R^{B,e}\right)\right)}{\rho_{B,I}}\times\beta E\left(R^{B,e}\right).$$

Accordingly,

$$\alpha_{I,B}=E\left(R^{I,e}\right)-\beta E\left(R^{B,e}\right)$$

$$>\left(\frac{sgn\left(E\left(R^{B,e}\right)\right)}{\rho_{B,I}}-1\right)\beta(E\left(R^{B,e}\right)$$

$$=\left(sgn\left(E\left(R^{B,e}\right)\right)-\rho_{B,I}\right)E\left(R^{B,e}\right)\frac{\beta}{\rho_{B,I}}. \tag{A.11}$$

We end the proof by showing that

$$\left(sgn\left(E\left(R^{B,e}\right)\right)-\rho_{B,I}\right)E\left(R^{B,e}\right)\frac{\beta}{\rho_{B,I}}>0. \tag{A.12}$$

To that end, we note that $E\left(R^{B,e}\right)-\rho_{B,I}$ has the same sign as $E\left(R^{B,e}\right)$, while β and $\rho_{B,I}$ have the same sign. This completes the proof. ■

Proof of Proposition 5. The result follows immediately from the fact that any informed excess return equals a linear combination of passive excess returns plus an additive constant, which is given by (22). Alternatively, one also can directly check that the style alpha satisfies the properties required of a performance functional g, with $g(1)=1$. ■

B. Numerical Examples

Example 1. Suppose that $K=2$, $\kappa=0.98$, and the information is captured by the assumption $p_{12}=p_{21}=\kappa$. We report results for two scenarios. In the first scenario, agents have CRRA preferences with risk aversion $\gamma=2$, dividends are independent and their logarithm is normally distributed with standard deviation equal to $\sigma=0.2$ and mean equal to $-\frac{1}{2}\sigma^2$. We apply Theorem 1b to compute equilibrium. In the second scenario, agents have CARA preferences with risk aversion $\gamma=2$ and dividends are normal and independent with standard deviation $\sigma=0.2$ and mean equal to 1. We apply Theorem 1 to compute the equilibrium.

Because of the symmetry of the setup, the price in both locations is the same ($P_1=P_2=P$) and portfolio holdings are symmetric, $X_{12}=X_{21}$

Table B2
Portfolio weights and alphas

Portfolio weights (pct.)						Passive alphas (pct.)	Active alphas (pct.)		
$\gamma=1$			$\gamma=3$						
49	24	18	39	31	29	0.23	2.27	0.43	0.23
36	58	03	34	40	25	−0.45	1.58	1.58	−0.24
15	18	79	26	29	46	0.23	0.43	0.23	2.27

The two tables under "portfolios" provide the percentage shares of risky assets invested by agent i in market j. Columns correspond to agents and rows to markets. Portfolios depend on whether $\gamma=1$ or $\gamma=3$. The column "passive alphas" provides the passive alphas obtained in the three different markets. The table "active alphas" provides the alphas obtained by (column) agent i in (row) market j. Both passive and active alphas do not depend on γ.

Example 3. Suppose that $K=3$, $\kappa=0.98$, $\gamma=4$, and the covariance and information matrices D_{ij} and p_{ij} are both symmetric and given by

$$\Omega=\begin{bmatrix} 0.096 & 0 & -0.036 \\ 0 & 0.06 & -0.03 \\ -0.036 & -0.03 & 0.03 \end{bmatrix},\ p_{ij}=\begin{bmatrix} 1 & 0.9980 & \kappa \\ 0.9980 & 1 & \kappa \\ \kappa & \kappa & 1 \end{bmatrix}.$$

Once we solve for an equilibrium, we obtain the vector of passive strategy alphas $\alpha_j=(2.45\%,\ 0.51\%,\ -2.64\%)$, whose smallest (largest) value is lower (higher) than $\kappa-1$ ($\kappa^{-1}-1$).

Example 4. Suppose that $K=3$, $\kappa=0.98$, $\gamma=1$, and the covariance matrix of D_{ij} and the information matrix for p_{ij} are both symmetric and given by

$$\Omega=\begin{bmatrix} 0.05 & 0.04 & -0.06 \\ 0.04 & 0.05 & -0.06 \\ -0.06 & -0.06 & 0.09 \end{bmatrix},\ p_{ij}=\begin{bmatrix} 1 & 0.998 & \kappa \\ 0.998 & 1 & \kappa \\ \kappa & \kappa & 1 \end{bmatrix}.$$

In equilibrium, agents 1 and 2 participate (i.e., have positive holdings) in all markets, whereas agent 3 holds a positive position only in market 3 and zero positions in markets 1 and 2. In this example, agent 3 has a CAPM alpha equal to -1.91% on her (optimal) portfolio, despite utilizing superior information in market 3.

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